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Structural Behavior of Battle-Deck Floors. *Inge Lyse and Ingvald E. Madsen*	Jan., 1938	
Discussion (Authors' closure)	June, Sept., 1938, Jan., 1939	Closed
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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

GRAPHICAL ARCH ANALYSIS APPLICABLE TO ARCH DAMS

By Carl H. Heilbron, Jr.,[1] Assoc. M. Am. Soc. C. E., and William H. Saylor,[2] Jun. Am. Soc. C. E.

Synopsis

In this paper is presented a system, mostly graphical, for analyzing unsymmetrical fixed-end arches of variable thickness and radius under loads of any type. The general equations of arch analysis are first written, from which the deflections and stresses in the arch may be obtained. These equations are then modified to include the concept of a "trial crown thrust," this concept making it possible to obtain arch stresses and deflections with all necessary accuracy when the greater part of the computations are performed graphically. The graphical constructions used in evaluating the equations are developed, the procedure is outlined in detail, and an illustrative example is given.

Introduction

In 1936 the Design Division of The Metropolitan Water District of Southern California undertook the design of the Gene Wash and Copper Basin Dams,[3] arch structures 156 and 210 ft high which form reservoirs near the easterly end of the Colorado River Aqueduct. In analyzing the designs for these dams, a system of trial load analysis was developed and successfully applied, following the principles presented by C. H. Howell, M. Am. Soc. C. E., and the late Mr. A. C. Jaquith.[4] A most important part of this system was the analysis of the arches. Disregarding the broader aspects of the analysis of arch dams, the details of the method of analyzing arches, as applied to this problem, are herein outlined.

Although, to date, the procedure presented has been used only for dam arches, it is equally applicable to arches of other types. It covers any fixed-end arch, symmetrical or otherwise, under any loading or temperature change.

Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **March 15, 1939.**

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[3] "High Arch Dams to Be Built on Colorado River Aqueduct," *Western Construction News*, March, 1937.

[4] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1191.

Deformations due to moment, thrust, shear, and uniform temperature change are included. Complications such as foundation deformation, interior arch sections, moment loads, and non-uniform temperature change, may be introduced.

The general theory of arch analysis will be reviewed briefly. The usual assumptions as to homogeneity of material in the arch and the application of Hooke's law are made. The arch is considered as cut at some point known as the crown. The effect of either half of the arch upon the other is then replaced by three unknown forces acting at the crown, a moment, M_c, and two direct forces, H and V. These three forces are applied to both halves of the arch, but in opposite directions, and each half of the arch is considered as a cantilever acted upon by its share of the external load, the temperature change, and the unknown crown forces. Equations are then written which give the three cantilever deflections at the crown (rotation, β_c, and translations, Δ_x and Δ_y). As the crown deflections are the same for each half of the arch, corresponding expressions for the two halves are equated, giving three formulas that may be solved simultaneously for the values of the crown forces. With the crown forces determined, each half of the arch is analyzed as a cantilever under a known system of loads.

The foregoing general approach to arch analysis has been presented in detail elsewhere[3] and in the following explanation it is assumed to be familiar to the reader. The equations for the crown deflections of an arch are given without derivation. Starting from this point, necessary changes are made in the equations, and the graphical construction used in this method of arch analysis is developed.

Notation.—The symbols used in this paper are defined where they first appear and are assembled for reference in the Appendix. The sign conventions adopted for the paper are either given in the definitions or illustrated by Fig. 1.

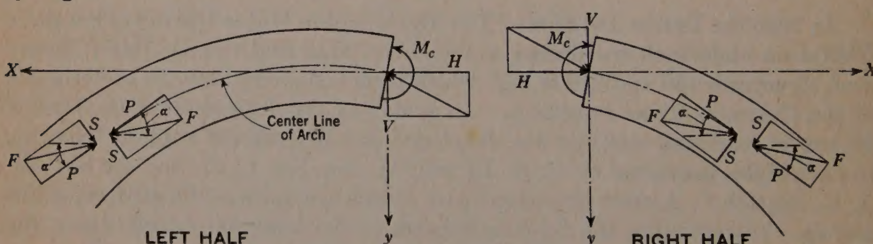

FIG. 1.—SIGN CONVENTIONS FOR ARCH ANALYSIS; POSITIVE QUANTITIES ILLUSTRATED

EQUATIONS

General equations for the crown deformations of an arch, including the effects of moment, thrust, shear, and temperature strain, were given by George R. Rich,[5] M. Am. Soc. C. E., in 1929. The crown deformations for the left half of the arch are:

Rotation at the crown:

$$\beta_c = + \sum_l \frac{M_e\, s}{E\, I} + \sum_l \frac{V\, x\, s}{E\, I} + \sum_l \frac{H\, y\, s}{E\, I} + \sum_l \frac{M_c\, s}{E\, I} \quad \text{.......} (1)$$

[5] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1230.

vertical deflection at the crown (positive upward):

$$\Delta_y = + \sum_l \frac{M_e\, x\, s}{E\, I} + \sum_l \frac{V\, x^2\, s}{E\, I} + \sum_l \frac{H\, x\, y\, s}{E\, I} + \sum_l \frac{M_c\, x\, s}{E\, I}$$

$$- \sum_l \frac{P_e\, s \sin\alpha}{E\, t} + \sum_l \frac{V\, s \sin^2\alpha}{E\, t} - \sum_l \frac{H\, s \sin\alpha \cos\alpha}{E\, t}$$

$$+ y_{al}\, \epsilon\, T + n \sum_l \frac{S_e\, s \cos\alpha}{E\, t} + n \sum_l \frac{V\, s \cos^2\alpha}{E\, t} + n \sum_l \frac{H\, s \sin\alpha \cos\alpha}{E\, t} \quad \ldots (2)$$

and, horizontal deflection at the crown (positive leftward):

$$\Delta_x = + \sum_l \frac{M_e\, y\, s}{E\, I} + \sum_l \frac{V\, x\, y\, s}{E\, I} + \sum_l \frac{H\, y^2\, s}{E\, I} + \sum_l \frac{M_c\, y\, s}{E\, I}$$

$$+ \sum_l \frac{P_e\, s \cos\alpha}{E\, t} - \sum_l \frac{V\, s \sin\alpha \cos\alpha}{E\, t} + \sum_l \frac{H\, s \cos^2\alpha}{E\, t}$$

$$- x_{al}\, \epsilon\, T + n \sum_l \frac{S_e\, s \sin\alpha}{E\, t} + n \sum_l \frac{V\, s \sin\alpha \cos\alpha}{E\, t} + n \sum_l \frac{H\, s \sin^2\alpha}{E\, t} \quad \ldots (3)$$

in which: β_c = the slope or rotation of the arch center line, at the crown (positive counter-clockwise); l = a subscript that refers to the left half of the arch; M_e = moment in the arch, taken about the center line, due to external loads; x and y = rectangular co-ordinates with their origin at the crown center line; s = length of a voussoir along the center line; E = modulus of elasticity in direct stress; I = moment of inertia of a radial section of arch; V = vertical force at the crown; H = horizontal force at the crown; M_c = moment in the arch at the crown; P_e = thrust in the arch parallel to the center line, due to external loads; α = angle between the X-axis and the center line of the arch; t = thickness of the arch, normal to the center line; x_a and y_a = values of x and y, respectively, for the abutment; T = temperature change (rise positive); S_e = shearing force due to external loads only; n = ratio of E to effective modulus in shear $\left(\text{that is, } \frac{E}{\frac{5}{6} E_s}\right)$; and, ϵ = thermal coefficient of expansion.

Similar equations may be written for the right side of the arch, with appropriate changes in signs.

It is convenient to write Equations (1), (2), and (3) for the left side of the arch in the following form:

$$E\, \beta_c = + A_{1l}\, M_c + B_{1l}\, H + C_{1l}\, V + D_{1l} \quad \ldots\ldots\ldots\ldots (4a)$$

$$E\, \Delta_y = + C_{1l}\, M_c + B_{2l}\, H + C_{2l}\, V + D_{2l} \quad \ldots\ldots\ldots\ldots (4b)$$

and,

$$E\, \Delta_x = + B_{1l}\, M_c + B_{3l}\, H + B_{2l}\, V + D_{3l} \quad \ldots\ldots\ldots\ldots (4c)$$

For the right side of the arch the equations are similar but with negative signs prefixed to the left sides of the equations.

Upon substituting $(1 - \sin^2\alpha)$ for $\cos^2\alpha$, and combining terms, the values of the coefficients, A, B, C, D, etc., are brought into the forms given in Table 1.

TABLE. 1.—VALUES OF THE COEFFICIENTS A, B, C, D, AND G

Coefficient	Quantity No.	Definition	Signs, at Right and Left Halves	
			Left	Right
A_1	1	$\Sigma \frac{s}{I}$	+	+
B_1	2	$\Sigma \frac{y\,s}{I}$	+	+
B_2	3	$\Sigma \frac{x\,y\,s}{I}$	+	−
	4	$(n-1)\,\Sigma \frac{s \sin \alpha \cos \alpha}{t}$	+	−
B_3	5	$\Sigma \frac{y^2\,s}{I}$	+	+
	6	$\Sigma \frac{s}{t}$	+	+
	7	$(n-1)\,\Sigma \frac{s \sin^2 \alpha}{t}$	+	+
C_1	8	$\Sigma \frac{x\,s}{I}$	+	−
C_2	9	$\Sigma \frac{x^2\,s}{I}$	+	+
	10	$n\,\Sigma \frac{s}{t}$	+	+
	11	$(n-1)\,\Sigma \frac{s \sin^2 \alpha}{t}$	−	−
D_1	12	$\Sigma \frac{M_e\,s}{I}$	+	+
D_2	13	$\Sigma \frac{M_e\,x\,s}{I}$	+	−
	14	$\Sigma \frac{P_e\,s \sin \alpha}{t}$	−	+
	15	$n\,\Sigma \frac{S_e\,s \cos \alpha}{t}$	+	−
	16	$y_a\,\epsilon\,T\,E$	+	−
D_3	17	$\Sigma \frac{M_e\,y\,s}{I}$	+	+
	18	$\Sigma \frac{P_e\,s \cos \alpha}{t}$	+	+
	19	$n\,\Sigma \frac{S_e\,s \sin \alpha}{t}$	+	+
	20	$x_a\,\epsilon\,T\,E$	−	−
G_1	21	$\Sigma \frac{M_t\,s}{I}$	+	+
G_2	22	$\Sigma \frac{M_t\,x\,s}{I}$	+	−
	23	$\Sigma \left(\frac{P_t\,s \sin \alpha}{t} - n \frac{S_t\,s \cos \alpha}{t} \right)$	−	+
	24	$y_a\,\epsilon\,T\,E$	+	−
G_3	25	$n\,\Sigma \frac{M_t\,y\,s}{I}$	+	+
	26	$\Sigma \left(\frac{P_t\,s \cos \alpha}{t} + n \frac{S_t\,s \sin \alpha}{t} \right)$	+	+
	27	$x_a\,\epsilon\,T\,E$	−	−

As an example of the interpretation of this table, the B_2-coefficients are written out in the usual algebraic form:

$$B_{2l} = + \sum \frac{x\,y\,s}{I} + (n - 1) \sum \frac{s \sin \alpha \cos \alpha}{t} \qquad (5a)$$

and,

$$B_{2r} = - \sum \frac{x\,y\,s}{I} - (n - 1) \sum \frac{s \sin \alpha \cos \alpha}{t} \qquad (5b)$$

The corresponding crown deflections for the two sides of the arch as given by Equations (4) are now equated. In the resulting expressions the coefficients for the left and right sides are combined to give new coefficients as follows:

$$A_1 = A_{1l} + A_{1r} \qquad (6a)$$

$$B_1 = B_{1l} + B_{1r} \qquad (6b)$$

$$C_1 = C_{1l} + C_{1r} \qquad (6c)$$

$$D_1 = D_{1l} + D_{1r} \qquad (6d)$$

and, similarly, for coefficients with the subscripts, 2 and 3. Using these new coefficients the three formulas resulting from equating the crown deflections are,

$$A_1\,M_c + B_1\,H + C_1\,V + D_1 = 0 \qquad (7a)$$

$$C_1\,M_c + B_2\,H + C_2\,V + D_2 = 0 \qquad (7b)$$

and,

$$B_1\,M_c + B_3\,H + B_2\,V + D_3 = 0 \qquad (7c)$$

When the A, B, C, and D coefficients are determined for any arch, their values may be substituted in Equations (7). Simultaneous solution of these three equations then gives the values of the crown forces, M_c, H, and V.

Trial Crown Force

In the system of analysis presented herein, Equations (7) are replaced by another similar set of formulas.

To show the need for this substitution, it will first be assumed that Equations (7) are being used. After the values of the crown forces have been determined from them, each half of the arch is considered as a cantilever acting under a known system of loads. From these loads, the moments, thrusts, and shears in the arch are obtained, and from the latter, in turn, the deflections are computed. It is found that in a comparatively thin arch most of the deflections are moment deflections. Thus, the accuracy with which the moments in the arch are found may largely determine the accuracy with which the deflections are found. The general equation for moment in the left half of an arch is,

$$M_l = M_c + H\,y + V\,x + M_{el} \qquad (8)$$

For the right half of the arch a similar equation holds, with a minus sign before the term, $V\,x$. With ordinary loads on a thin arch the value of M_{el} at any point will be negative and may be ten or a hundred times as large as the value of M_l. If the external load and the arch are reasonably symmetrical the values of M_c and $V\,x$ will be small, but the value of $H\,y$ will be positive and numerically about the same as the value of M_l. Thus, the value of M_l depends upon

the difference of two relatively large values, M_{el} and $H\,y$; and a small percentage error in either of these values will make a large percentage error in M_l. Using graphical procedures it has been found to be impracticable to evaluate these functions with enough accuracy to obtain a satisfactory determination of deflections for an arch.

To avoid this difficulty the concept of the "trial crown force" is introduced. The value of H is considered as divided into two parts,

$$H = H_t + H_c \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which H_t is the trial crown force. Its value is chosen at the beginning of an analysis, as close to H as it may be estimated; thereafter it is handled as if it were a part of the external load. Equations (7) may now be solved exactly as previously explained except that the value of H that is determined is not the true horizontal crown thrust but is H_c, that part of the crown thrust that was not included in the trial crown thrust, H_t. To avoid confusion, however, Equations (4), (7), and (8) are rewritten using a notation which includes the use of a trial crown thrust, H_t, as an external load. First Equation (8) becomes,

$$M_l = M_c + H_c\,y + V\,x + M_{tl} \quad\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

for the left half, and with $(-\,V\,x)$ for the right half. Symbol M_t is the moment at any point on the center line of the arch, due to the external loads and the trial crown force.

If H_t has been chosen close to the value of H, H_c will be small and M_{tl} will be of the same order as M_l. Thus, the accuracy obtainable with graphical procedure is sufficient to give a satisfactory determination of deflections in an arch, as will be shown subsequently.

Using the notation which includes the trial crown force, H_t, as an external load, Equations (4) become,

$$E\,\beta_c = +\,A_{1l}\,M_c + B_{1l}\,H_c + C_{1l}\,V + G_{1l} \quad\ldots\ldots\ldots (11a)$$

$$E\,\Delta_y = +\,C_{1l}\,M_c + B_{2l}\,H_c + C_{2l}\,V + G_{2l} \quad\ldots\ldots\ldots (11b)$$

and,

$$E\,\Delta_x = +\,B_{1l}\,M_c + B_{3l}\,H_c + B_{2l}\,V + G_{3l} \quad\ldots\ldots\ldots (11c)$$

and, similarly, with minus signs on the left, for the right side of the arch. The G-coefficients are the same as the D-coefficients except that M_e, P_e, and S_e are replaced by M_t, P_t, and S_t, in which P_t and S_t are thrust and shear due to external loads and trial crown force. The G-values are given in Table 1. They may be combined similarly to Equations (6), thus:

$$G_1 = G_{1l} + G_{1r} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and, similarly, for G_2 and G_3. Using the concepts and notation of Equations (11), Equations (7) become,

$$A_1\,M_c + B_1\,H_t + C_1\,V + G_1 = 0 \quad\ldots\ldots\ldots\ldots\ldots (13a)$$

$$C_1\,M_c + B_2\,H_t + C_2\,V + G_2 = 0 \quad\ldots\ldots\ldots\ldots\ldots (13b)$$

and,

$$B_1\,M_c + B_3\,H_t + B_2\,V + G_3 = 0 \quad\ldots\ldots\ldots\ldots\ldots (13c)$$

Equations (13) are the formulas that are used in determining the values of the crown forces, by a procedure which will be explained subsequently. The trial crown force need not be restricted to H. Under certain conditions the same concept may be applied beneficially to V or to M_c. The procedure is similar to that shown for H. Generally speaking, however, unless the arch shape or the load is very unsymmetrical, the use of a trial force for H is all that is desirable.

Deflection Equations

With the values of the crown forces, M_c, H_t, and V, known, the deflections in each half of the arch are calculated. The deflections of the arch crown may be determined by substituting the values of the crown forces into Equations (11); but the deflections at other points are also desired. These deflections are determined by means of the following relations: The rotation at any point of the arch is given by,

$$E\,\beta = \sum_a \frac{M\,s}{I} \qquad (14)$$

In any individual voussoir, the displacement of the end farther from the abutment with respect to the nearer end is composed of four parts, as follows:

Displacement due to moment (at right angles to the center line of the arch) is:

$$\Delta_M = \beta\,s \qquad (15a)$$

displacement due to thrust (parallel to the center line of the arch) is:

$$\Delta_P = \frac{P\,s}{E\,t} \qquad (15b)$$

in which P = the thrust parallel to the arch center line; displacement due to shear (at right angles to the center line of the arch) is:

$$\Delta_S = n\,\frac{S\,s}{E\,t} \qquad (15c)$$

in which S = the shearing force normal to the arch center line; and, displacement due to temperature (parallel to the center line of the arch) is:

$$\Delta_T = \epsilon\,T\,s \qquad (15d)$$

The displacement at any point may be expressed as the vector sum of a series of terms such as Equations (15). In a commonly used system of notation, a vector of length, r, in a direction making an angle, θ, with the positive X-axis is denoted by $r\,e^{i\theta}$, and the vector sum of two such vectors is indicated simply by using a plus sign.[6]

[6] See, for instance, "Engineer's Manual," by Ralph G. Hudson (1917), pp. 61–62.

In this notation, the deflection at any point on the left side of the arch can be found in magnitude and direction from,

$$E\,\Delta = \sum_a \left[E\,\beta\,s\,e^{i(\alpha+0.5\pi)} + \frac{P\,s}{t}\,e^{i(\alpha+\pi)} + n\,\frac{S\,s}{t}\,e^{i(\alpha+0.5\pi)} + \epsilon\,T\,E\,s\,e^{i\alpha}\right]\ldots(16)$$

and, similarly, for the right side, with appropriate changes in sign.

Finite Voussoirs

The terms in Table 1 have been written as summations of finite units. This is necessary as graphical constructions are to be used in the evaluation of the terms. Thus it is that the arch is considered divided into a number of finite lengths called voussoirs, each voussoir having a length, s. Any number of voussoirs may be chosen, but it is found that using from six to eight, of about equal length in each half of the arch, gives satisfactory results. The ends of the voussoirs are referred to as joints and are numbered from the crown to the abutments, left and right.

As finite voussoirs are used it is necessary to determine, in respect to each function, such as M, P, S, t, and I, whether it is to be measured at the joints or at the mid-points of the voussoirs. In this analysis the following convention is used: That part of the load on the arch which lies between the centers of adjacent voussoirs is considered as acting as a concentrated load at the joint. Thus, the thrust and shear are considered constant through the length of a voussoir. The length, s, entering into the $\frac{s}{t}$-terms, or thrust and shear terms, is measured from joint to joint, and the t-values are measured at the centers of the voussoirs. Increments of thrust and shear deflection are thus found from joint to joint.

As the change in moment values is obtained by multiplying the shear values by the length of the voussoir, the moments are found at the joints, and are considered as constant from center to center of adjacent voussoirs. In finding the $\frac{s}{I}$-values entering into the moment deflection terms, the s-values, therefore, are measured between voussoir centers, and the I-values are computed from t-values measured at the joints.

The $\frac{M\,s}{I}$-values, or curvature values, like the M-values, are considered as constant from center to center of voussoirs, giving increments of slope or rotation from center to center. Then rotations are found at the voussoir centers, and are considered as constant from joint to joint. Increments of moment deflections are thus found from joint to joint, just as for the thrust and shear deflections. All deflections, then, are found at the joints.

Graphical Multiplication and Integration

Several of the functions listed in Table 1 are evaluated by a method of graphical multiplication and integration. An example of this procedure

is given in Fig. 2, as taken from Figs. 3 and 4. The values of y and $\frac{s}{I}$ being given for each joint, it is desired to determine the values of $\sum \frac{y\,s}{I}$ and $\sum \frac{y^2\,s}{I}$ for the

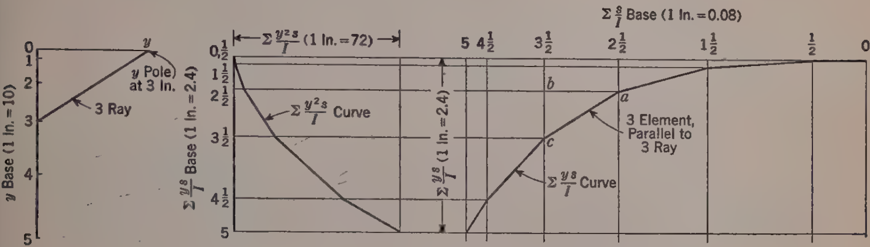

FIG. 2.—GRAPHICAL MULTIPLICATION AND INTEGRATION

left half of an arch. On a vertical line called the "y-base" (see Fig. 3(c)), the values of y are laid off to a given scale, each value being measured from the zero point. From an origin on a horizontal line called the "$\sum \frac{s}{I}$-base," values of $\frac{s}{I}$ (Fig. 3(d)) are laid off, successively, to another arbitrary scale. The abscissa of each point thus established then represents the value of $\sum \frac{s}{I}$ taken from the crown to some voussoir center, whose number is given to the point. Vertical lines are drawn through these $\sum \frac{s}{I}$-points, and are called by the same numbers.

A "y-pole" is established at a point an arbitrary horizontal distance ("pole distance") from the zero point of the y-base (Fig. 3(c)). Rays are considered as drawn from the y-pole to each of the points on the y-base. A broken line called the "$\sum \frac{y\,s}{I}$-curve" (Fig. 3(e)), is drawn as follows: Parallel to a ray from the y-pole to Point 1 on the y-base a line is drawn, called the "1 element," beginning at a point on the $\frac{1}{2}$ line of the $\sum \frac{s}{I}$-base and continuing to the $1\frac{1}{2}$ line. Parallel to a ray from the y-pole to the 2 point on the y-base is drawn a "2-element," beginning at the end of the 1-element and continuing to the $2\frac{1}{2}$-line of the $\sum \frac{s}{I}$-base. The procedure is continued to the abutment, the y-value at the abutment being used with the $\frac{s}{I}$-value for the last half voussoir. The projection on a vertical line of the jointed line thus drawn is the value of $\sum \frac{y\,s}{I}$ to a certain scale. The proof is as follows: Consider (Fig. 2) the 3-ray and the 3-element. Then, in the similar triangles $(y, 0, 3)$, and (a, b, c), $\frac{(y, 0)}{(0, 3)} = \frac{(a, b)}{(b, c)}$; whence,

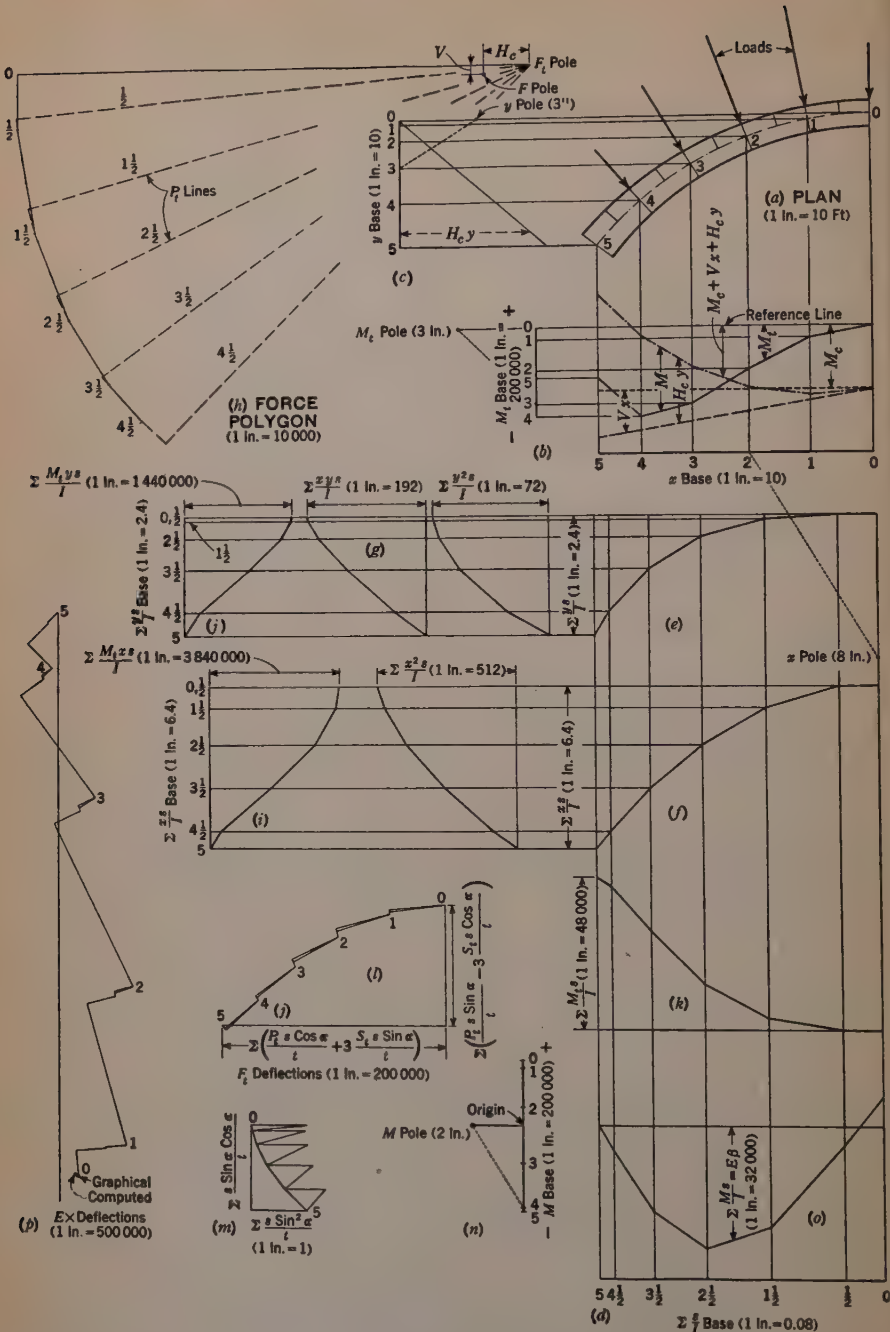

FIG. 3.—ANALYSIS OF LEFT HALF OF ARCH

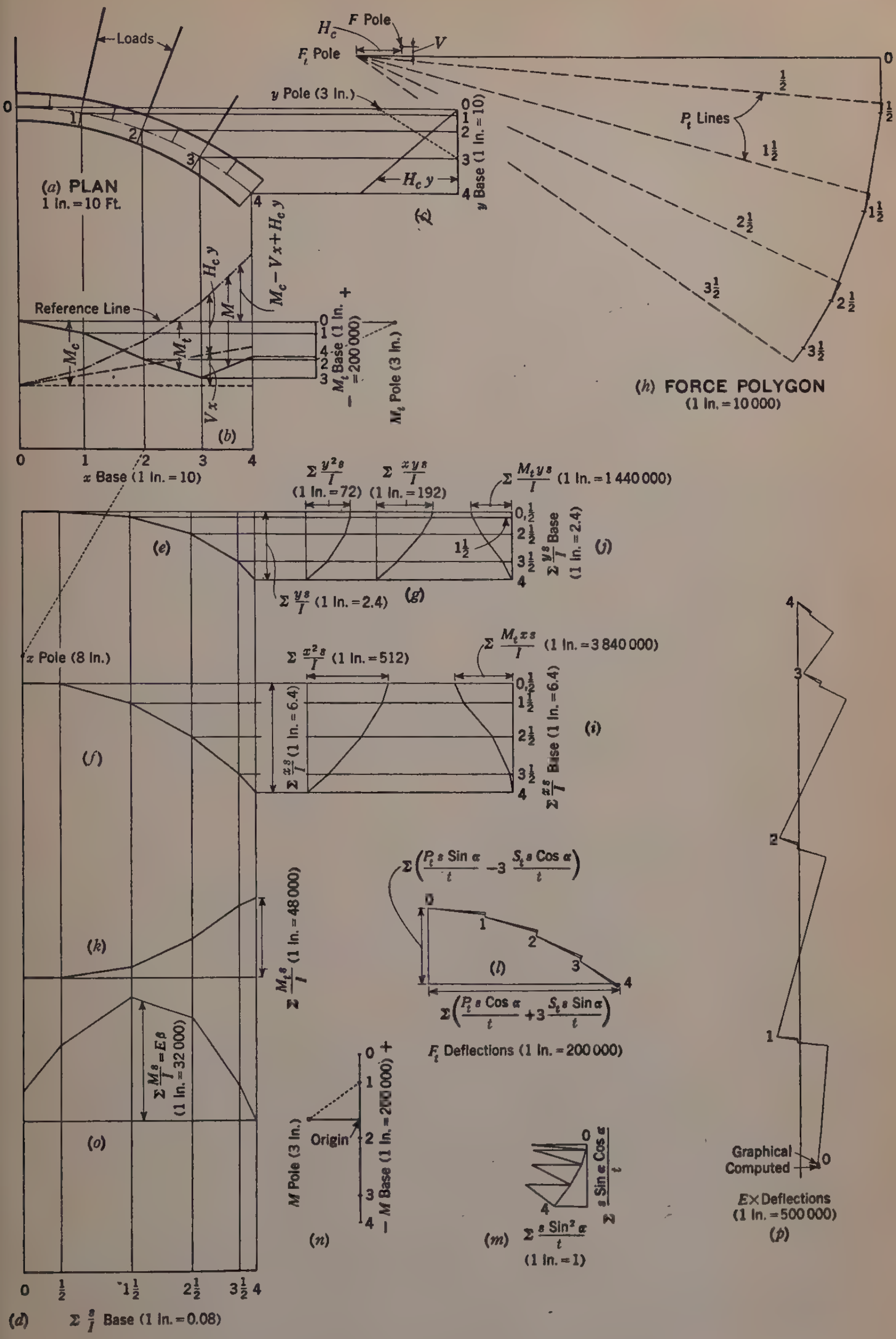

FIG. 4.—ANALYSIS OF RIGHT HALF OF ARCH

$(b, c) = \frac{(a, b)\,(0, 3)}{(y, 0)}$. The values of the three factors on the right are: $(y, 0)$ = pole distance (in.); $(0, 3) = \frac{y\text{-value of Point 3 (ft)}}{y\text{-scale}\left(\frac{\text{ft}}{\text{in.}}\right)}$; and,

$$(a, b) = \frac{\frac{s}{I} \text{ of arch length, } 2\tfrac{1}{2} - 3\tfrac{1}{2} \left(\frac{1}{\text{ft}^3}\right)}{\frac{s}{I}\text{-scale}\left(\frac{1}{\text{in.-ft}^3}\right)},$$

in which "y-scale $\left(\frac{\text{ft}}{\text{in.}}\right)$" denotes the number of feet of y corresponding to 1-in. on the diagram. Substitution gives,

$$(b, c) = \frac{\left[\frac{s}{I}\left(\frac{1}{\text{ft}^3}\right)\right][y \text{ (ft)}]}{\left[\frac{s}{I}\text{-scale}\left(\frac{1}{\text{in.-ft}^3}\right)\right]\left[y\text{-scale}\left(\frac{\text{ft}}{\text{in.}}\right)\right][\text{pole distance (in.)}]} \quad \ldots (17)$$

The interpretation of Equation (17) is that the vertical component of an element of the $\sum y \frac{s}{I}$-curve represents the product, $y \frac{s}{I}$, to a scale which is the continued product of the $\sum \frac{s}{I}$-scale, the y-scale, and the pole distance. Then, the vertical component of the entire $\sum \frac{y\,s}{I}$-line represents the value of $\sum \frac{y\,s}{I}$ to the same scale.

To obtain the value of $\sum \frac{y^2\,s}{I}$, a similar procedure is followed. As successive points of the $\sum \frac{y\,s}{I}$-curve are established, horizontal lines are drawn through them. These determine points on a vertical line called the $\sum \frac{y\,s}{I}$-base. A $\sum \frac{y^2\,s}{I}$-curve is drawn so that each element is perpendicular (instead of parallel) to the corresponding ray through the y-pole, and intersects the preceding and succeeding elements on the horizontal lines just mentioned. The horizontal component of the length of the $\sum \frac{y^2\,s}{I}$-curve represents the value of $\sum \frac{y^2\,s}{I}$ to a certain scale. Proof is analogous to the foregoing for the $\sum \frac{y\,s}{I}$-construction.

Other graphical procedures given in this paper are not as general in their use as the foregoing, and will be either self-explanatory or will be explained as their use is described.

METHOD OF ANALYSIS

In starting an analysis the loads on the arch, the temperature change in the arch, and the physical dimensions and characteristics of the arch, are assumed to be known. Furthermore, the factors governing foundation deformations are assumed to be known.

TABLE 2.—COMPUTATIONS FOR ILLUSTRATIVE EXAMPLE

Joint (see Figs. 3(a) and 4(a))	Thickness, t, in feet	Length, s, in feet	$\frac{s}{t}$	Length, s, in feet	$\frac{1}{I} = \frac{12}{t^3}$	$\frac{s}{I}$	Cumulative: $\Sigma \frac{s}{I}$	Load in pounds	Shear force, S_t, in pounds	Moments in Foot-Pounds	
										$S_t\,s$	M_t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0 l	10.8	....	....	12.0	0.00953	0.114	....	22 000	....	....	0
	10.8	24.0	2.22	....	....	....	0.114	....	−3 900	−93 600	....
1 l	10.9	....	....	24.0	0.00927	0.222	....	42 700	....	....	−93 600
	11.1	24.0	2.16	....	....	....	0.336	....	−10 200	−245 000	....
2 l	11.4	....	....	24.0	0.00810	0.194	....	37 700	....	....	−338 600
	11.8	24.0	2.03	....	....	....	0.530	....	−11 100	−266 500	....
3 l	12.2	....	....	24.0	0.00661	0.159	....	30 000	....	....	−605 100
	12.7	24.0	1.89	....	....	....	0.689	....	−4 100	−98 400	....
4 l	13.4	....	....	24.0	0.00499	0.120	....	20 000	....	....	−703 500
	14.1	24.0	1.70	....	....	....	0.809	....	+13 000	+312 000	....
5 l	14.9	....	....	12.0	0.00363	0.044	....	....	....	....	−391 500
			Σ = 10.00				0.853				
0 r	10.8	....	....	12.0	0.00953	0.114	....	22 000	....	....	0
	10.9	24.0	2.20	....	....	....	0.114	....	−3 900	−93 600	....
1 r	11.1	....	....	24.0	0.00877	0.210	....	41 000	....	....	−93 600
	11.4	24.0	2.11	....	....	....	0.324	....	−8 600	−206 500	....
2 r	11.7	....	....	24.0	0.00749	0.180	....	34 300	....	....	−300 100
	12.2	24.0	1.97	....	....	....	0.504	....	−6 100	−146 500	....
3 r	12.7	....	....	24.0	0.00586	0.141	....	23 500	....	....	−446 600
	13.3	24.0	1.80	....	....	....	0.645	....	+7 000	+168 000	....
4 r	14.1	....	....	12.0	0.00428	0.051	....		....	....	−278 600
			Σ = 8.08				0.696				

TABLE 2.—(*Continued*)

Joint (see Figs. 3(a) and 4(a))	Resultant force, F_t, in pounds	$\frac{F_t\,s}{t}$	$E\,\beta$	$E\,\beta\,s$	Resultant force, F, in pounds	$\frac{F\,s}{t}$	$\epsilon\,T\,E\,s$	Moment, M, in foot-pounds
(1)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
0 l	....	....	....	....	....	....	....	+503 000
	200 000	444 000	−22 000	−528 000	183 000	406 000	−581 000	....
1 l	....	....	....	....	....	....	....	+450 000
	201 000	434 000	−121 000	−2 904 000	184 500	399 000	−581 000	....
2 l	....	....	....	....	....	....	....	+145 000
	202 500	411 000	−149 000	−3 576 000	187 500	381 000	−581 000	....
3 l	....	....	....	....	....	....	....	−284 000
	204 500	387 000	−105 000	−2 520 000	189 500	358 000	−581 000	....
4 l	....	....	....	....	....	....	....	−626 000
	205 500	349 000	−29 000	−696 000	190 000	323 000	−581 000	....
5 l	....	....	....	....	....	....	....	−651 000
0 r	....	....	....	....	....	....	....	+503 000
	200 000	440 000	+93 000	+2 232 000	184 000	405 000	−581 000	....
1 r	....	....	....	....	....	....	....	+282 000
	201 000	424 000	+153 000	+3 672 000	187 000	395 000	−581 000	....
2 r	....	....	....	....	....	....	....	−145 000
	202 500	399 000	+127 000	+3 048 000	190 000	374 000	−581 000	....
3 r	....	....	....	....	....	....	....	−598 000
	203 500	366 000	+43 000	+1 032 000	191 000	344 000	−581 000	....
4 r	....	....	....	....	....	....	....	−812 000

To illustrate the procedure used in this method of arch analysis an example will be given in detail. For simplicity the illustration is confined to an unsymmetrical fixed-end arch without foundation deformations. The loads considered are restricted to direct forces in the plane of the arch, and a uniform temperature change is assumed. Without appreciable error, n is assumed equal to 3. At the end of this explanation, the manner in which other factors may be introduced will be indicated. It should also be noted that, in order to simplify the example, a smaller number of voussoirs are used than is generally desirable.

The arch chosen for this example is a section of the Gene Wash Dam, in California, at Elevation 710, and the loading is approximately that used at one stage of the trial-load analysis of that dam. All the graphical work of arch analysis is done on one large sheet (Figs. 3 and 4). The numerical work is contained in Tables 2 and 3 and in the solutions of the equations in Steps (29) and (30) presented subsequently herein. All quantities pertaining to the structure are given in units of feet and pounds and combinations of those units, exclusively, so that no further notation of dimensions is necessary. In the explanation that follows the graphical constructions referred to as *a*, *b*, *c*, etc., are to be found on Figs. 3 and 4. Steps that are followed separately for the two sides of the arch are described for the left side of the arch only.

Procedure

(1) In Construction (*a*) lay out the arch to a convenient scale and locate the center line. Establish a crown point.

(2) Along the center line of the arch, lay off voussoir lengths starting at the crown. Locate the centers of the voussoirs.

(3) Number the ends of the voussoirs along the center line, starting with 0 at the crown and numbering to the abutment. Assign whole numbers, $1\,l$, $2\,l$, etc., to the joints (ends of voussoirs) and half numbers, $\frac{1}{2}\,l$, $1\frac{1}{2}\,l$, etc., to the voussoirs themselves or to their centers. List the numbers in Column (1), Table 2.

(4) Scale the thickness of the arch at the center and ends of each voussoir and list in Column (2), Table 2.

(5) In Column (3), Table 2, list the voussoir lengths as measured along the center line of the arch.

(6) Calculate values of $\frac{s}{t}$ in Column (4), Table 2. Add these values to obtain the value of $\sum \frac{s}{t}$ and list this at the bottom of the column.

(7) Scale the distances along the center line of the arch between the centers of the voussoirs. At the crown and abutment, use the distance from crown or abutment to the center of the adjacent voussoir. List the distances in Column (5), Table 2.

(8) Calculate values of $\frac{1}{I} = \frac{12}{t^3}$ and $\frac{s}{I}$ in Columns (6) and (7), Table 2.

TABLE 3.—COMPUTATION OF COEFFICIENTS; ILLUSTRATIVE EXAMPLE

($\epsilon = 0.0000056$ per Degree Fahrenheit; $T = -10.0°$ F; $E = 432\,000\,000$; $\epsilon T E = -24\,200$; $n = 3$)

Quantity No. (see Table 1, Column (3))	Scale	Measured		Value		Coefficients	
		Left (inches)	Right (inches)	Left	Right	Symbol	Value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	....	....	....	+0.853	+0.696	A_{1l} = A_{1r} = A_1 =	+0.853 +0.696 +1.549
2	2.4	+4.69	+2.64	+11.26	+6.34	B_{1l} = B_{1r} = B_1 =	+11.26 + 6.34 +17.60
3 4	192 1	+4.53 +3.36	+2.15 +2.44	+870 +7	−413 −5	B_{2l} = B_{2r} = B_2 =	+877 −418 +459
5 6 7	72 1	+4.34 .... +2.16	+1.65 .... +1.22	+312.5 +10.0 +4.3	+118.8 +8.1 +2.4	B_{3l} = B_{3r} = B_3 =	+326.8 +129.3 +456.1
8	6.4	+6.33	+4.30	+40.5	−27.5	C_{1l} = C_{1r} = C_1 =	+40.5 −27.5 +13.0
9 10 11	512 1	+5.31 .... +2.16	+3.08 .... +1.22	+2 720 +30 −4	+1 577 +24 −2	C_{2l} = C_{2r} = C_2 =	+2 746 +1 599 +4 345
21	48 000	−5 92	−3.11	−284 000	−149 000	G_{1l} = G_{1r} = G_1 =	−284 000 −149 000 −433 000
22 23 24	3 840 000 200 000	−5.02 +4.72 +50.5	−2.23 +3.04 +33.2	−19 280 000 −944 000 −1 222 000	+8 565 000 +608 000 +803 000	G_{2l} = G_{2r} = G_2 =	−21 446 000 +9 976 000 −11 470 000
25 26 27	1 440 000 200 000	−4.20 +8.76 +104.5	−1.48 +7.40 +87.8	−6 050 000 +1 752 000 +2 530 000	−2 131 000 +1 480 000 +2 125 000	G_{3l} = G_{3r} = G_3 =	−1 768 000 +1 474 000 −294 000

(9) Starting at the crown, add progressively values of $\frac{s}{I}$ to obtain values of $\sum \frac{s}{I}$ in Column (8), Table 2.

(10) In Construction (*b*), establish a horizontal line, or "*x*-base," and in Construction (*c*), a vertical line, or "*y*-base." Through the ends of the voussoirs on the center line, draw vertical and horizontal lines, known as *x*-lines and *y*-lines. Number the intersections of these lines with the *x*-base and *y*-base to correspond with the numbers of the ends of the voussoirs.

(11) Locate x-poles and y-poles on perpendiculars through the zero points of the x-base and y-base, at distances that will give convenient sizes for the curves described subsequently. Record the pole distances, in inches.

(12) In Construction (d), plot a $\sum \frac{s}{I}$-base as follows: From a zero point lay off, horizontally to some convenient scale, values of $\sum \frac{s}{I}$ from Column (8), Table 2. Record the scale and number the points established with the corresponding voussoir numbers. Draw through each point a vertical line.

(13) In Construction (e), use the y-pole and y-base to plot a $\sum \frac{y\,s}{I}$-curve on the $\sum \frac{s}{I}$-base, and a $\sum \frac{y^2\,s}{I}$-curve on the $\sum \frac{y\,s}{I}$-base. This procedure is one of graphical multiplication and integration, and has been described previously.

(14) In Construction (f), use the x-pole and the x-base to plot $\sum \frac{x\,s}{I}$ on the $\sum \frac{s}{I}$-base, and $\sum \frac{x^2\,s}{I}$ on the $\sum \frac{x\,s}{I}$-base; and in Construction (g) plot $\sum \frac{x\,y\,s}{I}$ on the $\sum \frac{y\,s}{I}$-base. This procedure is similar to that of Step (13).

(15) In Column (9), Table 2, list the load considered to act at each joint. Take this as the total load actually acting between centers of the voussoirs; except that at the crown, take the load actually applied in the adjacent half-voussoir.

(16) Estimate a trial horizontal crown force, H_t, as by multiplying the average unit load on the arch by the estimated average radius of the thrust line. Apply the same crown thrust to each half of the arch.

(17) In Construction (h), draw a force polygon to a convenient scale. Include H_t and the loads of Column (9), Table 2. Start the polygon from a point called the F_t-pole. From this pole plot, horizontally, a vector representing H_t, and from the end of H_t plot, successively, vectors representing in magnitude and direction all the loads on the half-arch, beginning at the crown. Mark the points on this polygon with half-numbers, Point $1\frac{1}{2}$ being between the loads at Joints 1 and 2, etc.

(18) Through the F_t-pole draw a ray, or P_t-line, for each voussoir, in the direction of the thrust, P_t, in the voussoir. Take this direction as parallel to a line through the two center-line points at the ends of the voussoir.

(19) Obtain the average shear, S_t, in each voussoir by scaling the perpendicular distance from its P_t-line to its force polygon point. List these values in Column (10), Table 2. If the diagram is drawn as shown, call the shear, S_t, positive if the P_t-line is below the force polygon point.

(20) Multiply S_t by s of Column (3), Table 2, to obtain, in Column (11), the change in moment between the ends of each voussoir. Sum up from the crown toward the abutment to obtain values of M_t in Column (12).

(21) In Construction (b), draw a horizontal reference line intersecting the x-lines. From this line plot, to a convenient scale, values of M_t to form an M_t

versus x-curve. Plot positive values upward. To form an M_t-base, project the points on this curve horizontally on to a vertical line.

(22) Locate an M_t-pole on a perpendicular through the zero point of the M_t-base, at a convenient distance. Record this pole distance, in inches.

(23) Using the M_t-base and M_t-pole, plot: $\sum \frac{M_t x s}{I}$ on the $\sum \frac{x s}{I}$-base (Construction (i)); $\sum \frac{M_t y s}{I}$ on the $\sum \frac{y s}{I}$-base (Construction (j)); and, $\sum \frac{M_t s}{I}$ on the $\sum \frac{s}{I}$-base (Construction (k))—all by the method of "graphical multiplication and integration."

(24) Find values of F_t, the resultant force at any point due to external loads and the trial crown force, by scaling, on the force polygon (Construction (h)), from the F_t-pole to each numbered point ($\frac{1}{2}$, $1\frac{1}{2}$, etc.). List the values in Column (13), Table 2. Multiply by $\frac{s}{t}$ of Column (4) and list the products, $\frac{F_t s}{t}$, in Column (14).

(25) In Construction (l), draw an "F_t-deflection diagram," an element of which is illustrated in Fig. 5. Proceed as follows: To some convenient scale plot

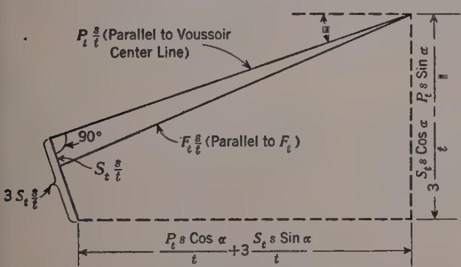

FIG. 5.—F_t DEFLECTION DIAGRAM

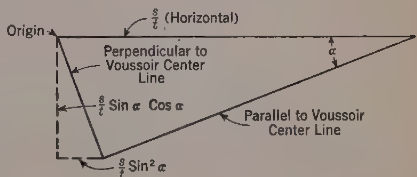

FIG. 6.—CONSTRUCTION FOR FUNCTIONS, $\Sigma \frac{s}{t} \sin \alpha \cos \alpha$ and $\Sigma \frac{s}{t} \sin^2 \alpha$

the value of $\frac{F_t s}{t}$ for the first voussoir in the direction of F_t as taken from the force polygon; that is, from the F_t-pole to the point on the polygon corresponding to the voussoir. Resolve this vector into its $\frac{P_t s}{t}$ and $\frac{S_t s}{t}$-components, the first parallel, and the second normal, to the direction of the center line of the voussoir, as taken from the corresponding P_t-line on the force polygon. Increase the $\frac{S_t s}{t}$-component to n, or three times its length, the increase being away from the $\frac{P_t s}{t}$-component. From the end of this increased normal, draw the $\frac{F_t s}{t}$-line for the next voussoir in the same manner, and repeat the procedure. Continue to the abutment. The horizontal and vertical distances from the

beginning to the end of this construction give, respectively, the values of Quantities Nos. 26 and 23, Table 1, as required for the coefficients, G_3 and G_2. This may be verified by a consideration of Fig. 5.

(26) To obtain the values of Quantities Nos. 4 and 7, Table 1, make Construction (m), an element of which is illustrated in Fig. 6. Proceed as follows: From Column (4), Table 2, take the value of $\frac{s}{t}$ for the first voussoir and plot it as a horizontal line on a convenient scale to the right from the point of origin. From the right end of this $\frac{s}{t}$-line draw a line parallel to the center line of the voussoir, and from the origin end draw a line perpendicular to the center line of the voussoir. Using the intersection of the two lines as the origin, repeat the construction for the next voussoir, and continue to the abutment. Take the vertical and horizontal components of the distance from beginning to end of this construction as the required summations. Proof is evident on inspection of the diagram.

(27) At the top of Table 3 list the values of ϵ, T, E, and the value of the product, $\epsilon\, T\, E$, with the same sign as T.

(28) Having now made all the graphical constructions for evaluating the terms listed in Table 1, determine their values and assemble them into the coefficients of Equations (11) and (13) by filling out Table 3. Fill out all lines of the table except Quantities Nos. 1, 6, 10, 24, and 27, as follows: Determine the scale factors for each function in number of units per inch, and list them in Column (2), Table 3. For the moment terms $\left(\text{which are those containing } \frac{s}{I}\right)$ take the scale factors as the product of the appropriate base scales and pole distances, as explained previously (see heading, "Graphical Multiplication and Integration"). For the other terms, take simply the scale used in the graphical work.

Measure off on Figs. 3 and 4 the values, in inches, of each function there determined and list it in the appropriate line of Columns (3) and (4), in Table 3. Prefix to each value, as measured, its proper sign. If, in Figs. 3 and 4, the positive direction of each term introduced has been taken as shown or specified, and if the relative positions of poles and bases are as shown, determine the signs as follows: Take the sign of any function, except that of Quantity No. 26, Table 3, as plus if its value is measured from the abutment end of the curve up or to the left for the left half of the arch, and up or to the right for the right half of the arch. For the function of Quantity No. 26, take the reverse.

In Columns (5) and (6), Table 3, place the product of the "scale" values and the "measured" values, and in Quantities Nos. 4, 11, and 7, also multiply by the factor, $(n - 1)$, or by 2. Determine the sign of each term by multiplying the sign in the "measured" column by the sign in Table 1.

In Quantity No. 1, Columns (5) and (6), Table 3, place the values of $\sum \frac{s}{I}$ as taken from Column (8), Table 2. In Quantity No. 6, Columns (5) and (6), place the values of $\sum \frac{s}{t}$ as taken from Column (4), Table 2; and in Quantity

No. 10, Columns (5) and (6), place the same values multiplied by n, or by 3. The signs are always positive.

From Construction (a) scale the values of x_a and y_a, and list them in Columns (3) and (4) of Quantities Nos. 27 and 24, respectively. Prefix the proper signs. Place the products of these quantities and $\epsilon\, T\, E$ in Columns (5) and (6), Table 3. Prefix the sign determined by multiplying the signs of the two factors and the sign given in Table 1. In Column (8), Table 3, add algebraically the proper terms of Columns (5) and (6) to determine the coefficients of Equations (11) and (13).

(29) Substitute the values of the coefficients just found into Equations (13). Solve the three equations simultaneously for M_c, H_c, and V:

$$\begin{aligned}
&+\ 1.55\, M_c + 17.60\, H_c + 13\, V - 433\,000 = 0\\
&+ 13.0\ \ M_c + 459.0\ \ H_c + 4\,345\, V - 11\,470\,000 = 0\\
&+ 17.6\ \ M_c + 456.1\ \ H_c + 459\, V - 294\,000 = 0
\end{aligned}$$

The solution is: $M_c = +\,503\,050$; $H_c = -\,22\,277$; and $V = +\,3\,488$.

(30) Substitute the values of the coefficients for the left half of the arch from Table 3 into Equations (11) to obtain the values of the crown deflections: $E\,\beta_c = +\,36\,000$; $E\,\Delta_y = -\,11\,031\,000$; $E\,\Delta_x = -\,325\,000$. The same deflections may be computed for the right half of the arch.

This completes the first part of the arch analysis. The following procedure evaluates the deflections at all points in the arch and determines the arch stresses.

(31) On the y-base (Construction (c)), plot values of $H_c\, y$ to the scale used for M_t in Step (21), as follows: Compute $H_c\, y$ for any point; plot its value horizontally from the corresponding point on the y-base to the scale of M_t; and through this point and the zero point of the y-base draw a straight line. The intercepts between this line and the y-base are the required $H_c\, y$-values at all points.

(32) Construct a curve of values of $(M_c \pm V\, x + H_c\, y)$, *versus* x, with the same scale and the same reference or zero line as the M_t-curve drawn in Step (21), but with reversed sign, as follows: From the reference line measure, vertically, the quantity, M_c, and draw a horizontal line. This is the M_c-line. Through the intersection of the M_c-line with the line, $x = 0$, draw a line with slopes such that intercepts on the x-line between it and the M_c-line give the values of $V\, x$. The procedure is similar to that used in Step (31) for finding $H_c\, y$. From the diagram of Step (31), take off values of $H_c\, y$ with dividers and lay them off along the corresponding x-lines from the line just drawn. Connect the points thus found, forming the desired curve.

In the foregoing procedure some confusion may arise as to the direction in which the constructions are made. As stated previously, the signs of the construction are reversed from that of Step (21). Thus, a positive M_c is plotted down, a positive $V\, x$ is plotted down on the left and up on the right, and a positive $H_c\, y$ is plotted down. By plotting the values of $M_c + H_c\, y \pm V\, x$ to the same scale and from the same reference line as M_t, but in the opposite direction, the intercepts on the x-lines between the two curves give the value,

$M = M_t + M_c + H_c\,y \pm V\,x$ (Equation (10)). The value, M, is positive if measured up from the last constructed curve.

(33) Establish a vertical line as an M-base (Construction (n)). Take off values of M with dividers from the construction of Step (32), and lay them off on the M-base from some reference point or origin, plotting positive values upward. Also, scale off values of M and list them in Column (20), Table 2.

(34) Locate an M-pole on a perpendicular through the origin of the M-base, at a convenient distance. Record this pole distance.

(35) In Construction (o), use the M-pole and M-base, to plot $\sum \frac{M\,s}{I} = E\,\beta$ on the $\sum \frac{s}{I}$-base, using the method of graphical multiplication and integration previously explained. Start the diagram at both abutments and work toward the crown, where it should close. With the sign convention on the M-base as stated and the relative pole positions as shown, the value of $\sum \frac{M\,s}{I}$ or $E\,\beta$ will be positive upward. Determine the scale of $E\,\beta$ from the product of the base scales and pole distance.

(36) Scale the values of $E\,\beta$ at each mid-voussoir point on the diagram just drawn and enter them in Column (15), Table 2. Multiply by values of s from Column (3) and enter the product $E\,\beta\,s$ in Column (16).

(37) Add, vectorially, the values of H_c and V, determined in Step (29), into the force polygon (Construction (*h*)). To do this, plot, successively, from the F_t-pole, vectors representing in magnitude and direction the values of H_c and V but with reversed sign. That is, plot positive values of H_c horizontally to the right and positive values of V vertically downward on the left-hand polygon, and in the reverse directions on the right-hand polygon. The new pole thus established is the F-pole, and the force polygon from this pole represents the true forces existing in the arch.

(38) Find values of F, the resultant force at any point in the arch, by scaling, on the force polygon, from the F-pole to the numbered points ($\frac{1}{2}$, $1\frac{1}{2}$, etc.) and list them in Column (17), Table 2. Multiply by $\frac{s}{t}$ from Column (4) and enter Product $\frac{F\,s}{t}$ in Column (18).

(39) Multiply s, Column (3), by $\epsilon\,T\,E$ of Step (27) and enter the product, $\epsilon\,T\,E\,s$, in Column (19).

(40) In Construction (p), plot a deflection diagram to a convenient scale, using the vectors indicated in Equation (16). Beginning at some origin plot successively, for each voussoir from abutment to crown, its four increments of deflection. This procedure is illustrated in Fig. 7 for Voussoir $2\frac{1}{2}\,l$. First, plot a vector representing the value, $\frac{F\,s}{t}$, taking the value from Column (18), Table 2, and the direction the same as F, taken from the force polygon. Resolve this vector into its $\frac{P\,s}{t}$ and $\frac{S\,s}{t}$-components parallel and perpendicular to

the voussoir center line, and increase the vector, $\frac{S\,s}{t}$, by three times in the direction away from $\frac{P\,s}{t}$. This procedure is similar to that described in Step (25). From the end of the foregoing construction plot a vector representing, in magnitude and direction, the function, $\epsilon\, T\, E\, s$. This vector is plotted parallel to the arch center line, in the direction away from the abutment if it is positive. Magnitudes are given in Column (19), Table 2. Finally, plot a vector representing, in magnitude and direction, the value of the moment deflection, $E\,\beta\,s$. This vector is plotted perpendicular to the arch center line, a positive value being toward the arch extrados on the left and toward the arch intrados on the right. Magnitudes are given in Column (16), Table 2. Number the points on this diagram the same as the corresponding joint numbers. Plot the crown displacements as computed in Step (30) to check the results of this step.

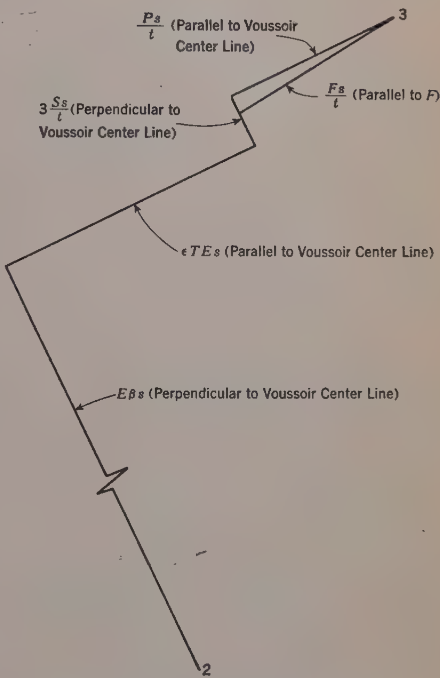

FIG. 7.—CONSTRUCTION OF ELASTICITY DEFLECTION DIAGRAM

(41) Calculate stresses. Obtain values of P from the force polygon as follows: For the abutment point take the component in the direction of the arch axis of the distance from the F-pole to the abutment point. For any other joint, obtain P-components for adjacent mid-voussoir points on either side, and interpolate. Obtain M-values from Column (20) and t-values from Column 2, Table 2. The stresses are computed at the desired points with the familiar equations. No example is given.

ADDITIONAL FACTORS

The foregoing simplified example illustrates all the fundamental points of this system of analysis. In order to demonstrate the wide range of applicability of the method, the procedures used in allowing for several other factors will be described briefly. However, before proceeding to these complicating factors, two possible simplifications will be presented. The first concerns the treatment of shear deformations. In thin arches, these deformations are negligible and may very well be omitted. Instead of omitting them entirely, however, it is simpler to assume that the effective shear modulus is equal to E, or that n equals 1. Then, the total deformation due to thrust and shear is always in the direction of the resultant force producing it. In Table 1 the terms with

coefficients, $(n - 1)$, drop out. In Step (25), since the $\frac{S_t s}{t}$-components are not to be multiplied by 3, it is unnecessary to resolve the $\frac{F_t s}{t}$-values into components; they are simply plotted, successively, and added vectorially. Since the terms obtained in Step (26) do not appear, this step is omitted. Finally, in Step (40), as in Step (25), the $\frac{F s}{t}$-values are not resolved into components.

The second simplification occurs where the arch shape is symmetrical. Then many of the procedures require solution for one side of the arch only. The work is simplified still further if the loads are also symmetrical. In this case, one side only needs to be considered in all the procedures.

Interior or Net Arches.—In the analysis of a plain concrete arch, such as in a dam, it is often desirable to assume that the concrete is unable to take tension, and will crack instead. If, from a previous analysis, the area subject to tension can be determined approximately, this area can be considered as not acting structurally. Only the remaining interior arch or net section is then used in the analysis. No change in procedure is involved.

Moment Loads.—After a few trials in a trial-load analysis, approximate values of twisting moments in the cantilevers can be determined, and, from these values, moment loads to be applied to arches can be found. These loads are applied by adding them to the values of Column (12), Table 2, to obtain new M_t-values.

Foundation Deformations.—In the analysis of the Gene Wash and Copper Basin Dams deformation effects in the foundation were generally included. Methods of calculating deformation factors have been presented elsewhere[7] and will not be discussed in this paper. However, it may be stated that these effects are included by considering an effective "abutment $\frac{s}{I}$" and an "abutment $\frac{s}{t}$," whose values are included in Columns (7) and (4), respectively, of Table 2, and are handled similarly to the other values in the same columns.

Non-Uniform Temperature.—A dam with its reservoir full of water will have a relatively constant temperature at the up-stream face and may have a quite different temperature at the down-stream face. This results in a temperature gradient, $\frac{dT}{dt}$, in the up-stream-down-stream direction, which may be quite important. To include this effect in the arch analysis the quantity, $\frac{dT}{dt} \epsilon E I$, is computed at each joint and added to M_t of Column (12), Table 2, to obtain a new M_t. These corrected M_t-values are used to determine crown forces and all deflections. In determining stresses the $\frac{dT}{dt} \epsilon E I$-corrections are subtracted from the final M-values.

The foregoing discussion of the inclusion of additional factors into the procedure shows that this method of analysis is readily adaptable to various condi-

[7] "Ueber die Berechnung der Fundamentdeformation," by Fredrik Vogt, Assoc. M. Am. Soc. C. E., Math. Naturv. Klasse 1925, No. 2.

tions. In general, it may be stated that any effect that can be included in any type of arch analysis may be included in this procedure. In many details of the procedure, alternative methods are available. Many of these methods have been tried, and some have been found equally as convenient as those adopted, but none was more so.

Conclusion

The procedure described herein was used about a hundred times in the course of the analyses of the Gene Wash and Copper Basin Dams in California. When first developed it was used at the same time as an entirely numerical method of computation, and a comparison of the two forms of analysis was possible. The graphical procedure gave accuracy comparable to that of the numerical method, and in both cases some of the work was done by men who followed the mechanical procedure given them, with little or no concern for the principles involved.

Several definite advantages were apparent for the graphical method. In the hands of an experienced operator the graphical procedure has some advantage as to time required. More important, however, is the fact that the graphical procedure presents to the calculator a picture of what has been done. When errors are made they often appear as discontinuities in curves, or as curves of disproportionate size or shape, and, therefore, are discovered much more readily than in the arithmetical method. The picture which the graphical work presents to the calculator also has another value. In an arch which is repeatedly analyzed for varying conditions, all the changes that occur in the component curves of the analysis have a significance to the analyst. For instance, in a case where changes in loads are being made to obtain, in successive analyses, closer approach to desired deflections, he is aided in judging as to whether the changes in assumptions are being chosen properly or what further changes should be made. Thus, whereas a numerical procedure tends to become an endless accumulation of meaningless data, the graphical procedure gives a concise picture of all the component parts of the work.

If different physical characteristics are used for each trial analysis, as when using net sections, a numerical procedure is likely to be very ponderous. The tendency of designers, therefore, has been to avoid the use of net sections or to make simplifying assumptions so as to bring the amount of work within practical limits. As demonstrated in this paper, however, new physical properties of the arch may be accepted, with this method, for each new arch analysis without unduly increasing the work. Thus, this graphical procedure may make practical an exact analysis which by other methods would be too cumbersome.

The concept of the trial crown force is the one important feature that has permitted the development of this method in which satisfactory accuracy is obtained with the extensive use of graphics. It should be noted that this concept would also allow the use of a procedure closely paralleling that presented, in which graphical work would be replaced by arithmetic, and slide-rule accuracy would be sufficient. For a person preferring numerical work to graphical work such a simplification would be of great help.

Acknowledgment

The procedures described in this paper were developed and used in the Design Office of The Metropolitan Water District of Southern California, under the direction of Julian Hinds, M. Am. Soc. C. E., Assistant Chief Engineer. All engineering and construction activities of the District are under the direction of Frank E. Weymouth, Hon. M. Am. Soc. C. E., General Manager and Chief Engineer.

APPENDIX

Notation

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials[8] compiled by a Committee of the American Standards Association with Society representation, and approved by the Association in 1932. The sign conventions are either given in the definitions or illustrated by Fig. 1.

A = coefficients in the general arch equations (see Coefficients A, B, C, D, and G, in Table 1);
a = a symbol referring to the abutments; generally used as a subscript;
B = coefficients (see Symbol A);
C = coefficients (see Symbol A);
D = coefficients (see Symbol A);
E = modulus of elasticity in direct stress; E_s = modulus of elasticity in shear;
e = a subscript denoting "due to external loads";
F = a resultant force at any point in an arch; F_t = a resultant force at any point in an arch due to external loads and a trial crown force;
G = coefficients (see Symbol A);
H = a "horizontal" force at the crown; that is, a force parallel to the X-axis; H_c = difference between the estimated and actual values of H at the crown; H_t = the estimated, trial, horizontal crown force;
I = moment of inertia of radial section of an arch;
l = a subscript denoting "the left half of the arch";
M = the moment in an arch at any point, taken about the center line of the arch; M_c = value of M at the crown; M_e = value of M at any point due to external loads; M_t = value of M at any point due to external loads and a trial crown force;
n = ratio of E to effective modulus in shear: $\frac{E}{\frac{5}{6} E_s}$;
P = thrust acting on an arch section, parallel to the center line; P_e = value of P due to external loads; P_t = value of P due to external loads and a trial crown force;
r = length of a vector; as a subscript, r denotes "the right half of the arch";

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[8] A. S. A.–Z 10A–1932.

S = a shearing force in a plane normal to the center line; S_e = value of S due to external loads only; S_t = value of S due to external loads and a trial crown force;
s = length of a voussoir, measured along the center line;
T = temperature change (rise positive);
t = thickness of the arch, normal to the center line; as a subscript, t denotes "due to external loads and a trial crown force";
V = "vertical" force at the crown; that is, a force parallel to the Y-axis;
x = a distance measured parallel to the X-axis, beginning at the crown center line; the X-axis is called "horizontal" and is approximately parallel to the arch center line at the crown; x_a = values of x for the abutment;
y = a distance measured parallel to the Y-axis, beginning at the crown center line; the Y-axis is called "vertical" and the positive y-direction is called "downward"; y_a = values of y for the abutment;
α = angle between the X-axis and the arch center line;
β = slope, or rotation, of the arch center line (counter-clockwise positive); β_c = the value of β at the crown;
Δ = vector displacement at any point on the arch center line; Δ_x = value of Δ at the crown in the X-direction (positive leftward); Δ_y = value of Δ at the crown in the Y-direction (positive upward);
ϵ = coefficient of thermal expansion;
θ = angular distance; angle between the X-axis and the direction of any vector (sign convention same as for α).

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

ENGINEERING GEOLOGY PROBLEMS AT CONCHAS DAM, NEW MEXICO

BY IRVING B. CROSBY,[1] AFFILIATE, AM. SOC. C. E.

SYNOPSIS

None of the varied types of foundation conditions of large dams presents more difficult problems than those of dams on uncemented shale or siltstone, and perhaps in no other branch of Engineering Geology is there more need of additional research. The principal purpose of this paper is to describe some of these problems, using Conchas Dam as an example, to explain how they were investigated and met in this particular instance, and to outline the research necessary to solve similar problems more adequately in the future. In addition, this paper outlines some of the problems encountered with high dams on sandstone containing water under artesian pressure, since Conchas Dam rests on both shale and sandstone.

INTRODUCTION

Conchas Dam is a project of the United States Army Engineers on the South Canadian River 30 miles northwest of Tucumcari, N. Mex., designed to regulate and utilize the flood waters of the river. The dam is nearly a quarter of a mile below the mouth of the Conchas River, at a point where the South Canadian River turns from a southerly to an easterly course. The length of the river above its junction with the North Canadian River is 700 miles, of which only 150 miles are in New Mexico. Its basin is wider in the upper part, however, with the result that 51% of the drainage area and all the major tributaries are in New Mexico. Flash floods of great magnitude are common, but during much of the year the river is nearly dry. The greatest recorded flood had a peak flow of 279 000 cu ft per sec at the dam site. The climate is semi-arid, the annual rainfall being less than 15 in., but the river has its sources in high mountains where the precipitation is somewhat greater.

The main dam is a gravity, concrete structure 240 ft high and 1 250 ft long with the crest of the parapet at Elevation 4 240, and an uncontrolled

NOTE.—Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **March 15, 1939.**

[1] Cons. Eng. Geologist, Boston, Mass.

over-fall spillway 340 ft long with a crest elevation of 4 201 ft. Earth wing-dams connect with both ends of the main dam and the South Dike extends 6 400 ft from the South Wing-Dam (see Fig. 1). In addition, north of the

FIG. 1.—CONCHAS DAM AND DIKES

Main Dam there is a concrete Emergency Spillway 3 000 ft long with earth dikes at either end. At the south end of the South Dike are the Irrigation Headworks and beyond is a small dike in a saddle. The earth wing-dams and dikes, the highest of which will be 96 ft, total 3 miles in length.

FIG. 2.—VIEW OF MAIN DAM, FACING UP STREAM, DECEMBER 24, 1937

The main concrete dam crosses the canyon of the river and rises above the walls of the canyon, the wing-dams extending to higher ground on either side (see Figs. 2 and 3). The main dam contains approximately 750 000 cu yd of concrete and the emergency spillway and irrigation headworks contain nearly 80 000 cu yd of additional concrete, making a total of 830 000 cu yd. The wing-dams and dikes contain approximately 2 900 000 cu yd of earth-fill and 790 000 cu yd of rock-fill.

When full, the reservoir extends about 14 miles up the South Canadian River and 11 miles up the Conchas River, but the valley of the Conchas River provides the greater part of the storage capacity due to its greater width. Above the dam site the South Canadian River is generally in a narrow canyon from 100 to 200 ft deep. The total storage capacity will be approximately 600 000 acre-ft. Of this, 100 000 acre-ft is dead storage, 300 000 acre-ft is irrigation storage, and 200 000 acre-ft, flood control storage.

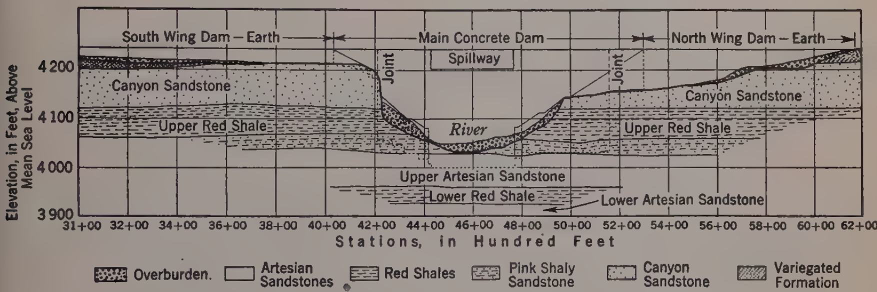

Fig. 3.—Geological Section Across Canyon on Line of Conchas Dam

Preliminary work was begun with Government forces in August, 1935, and a contract was let for the main dam and wing-dams on October 6, 1936. Additional information about the purpose of the project, its history, the engineering and construction features, and the hydrology can be found in papers by John R. Noyes,[2] Assoc. M. Am. Soc. C. E., and by Gerard H. Matthes,[3] M. Am. Soc. C. E.

The project has been in charge of Hans Kramer, M. Am. Soc. C. E., District Engineer, Conchas District. James H. Stratton, Assoc. M. Am. Soc. C. E., was Chief of the Engineering Division until May 15, 1937, when he was succeeded by Captain L. H. Foote. J. B. Alexander, Assoc. M. Am. Soc. C. E., has been Planning Chief and H. V. Pittman, M. Am. Soc. C. E., Construction Chief. Joel D. Justin, W. H. McAlpine, William Gerig, H. T. Cory, Gerard H. Matthes, Members, Am. Soc. C. E., and the late Louis C. Hill, Past-President, Am. Soc. C. E., were Consulting Engineers, and the writer was Consulting Geologist.

Geology of the Dam Site

Conchas Dam is in an extensive area of flat-lying sedimentary rocks known as the Red Beds. The rocks at the site are members of the Dockum

[2] "Conchas Dam and Reservoir Project," by John R. Noyes, *Engineering News-Record*, April 15, 1937, pp. 541–545.

[3] "Problems at Conchas Dam," by Gerard H. Matthes, *Civil Engineering*, July, 1936, pp. 437–441.

Group of Triassic Age. For convenience, the different members of this formation at the dam site were given local, descriptive names which are, beginning with the youngest and highest:

(1) Townsite Formation (sandstones and shales about 450 ft thick);
(2) Variegated Formation (shales with some sandstone about 300 ft thick);
(3) Canyon Sandstone (about 100 ft thick where not eroded);
(4) Upper Red Shale (average thickness, 64 ft);
(5) Pink Shaly Sandstone (average thickness, 25 ft);
(6) Upper Artesian Sandstone (average thickness, 64 ft);
(7) Lower Red Shale (average thickness, 30 ft); and,
(8) Lower Artesian Sandstone (thickness greater than 100 ft).

The canyon sandstone, upper red shale, and pink shaly sandstone outcrop at the main dam site, and the upper artesian sandstone was uncovered in the excavations. The lower red shale and the lower artesian sandstone underlie the dam site, and the South Dike is on the variegated formation.

The canyon sandstone is the rim rock of the canyon and, in general, is a hard grayish to brownish, massive sandstone; some parts of it, however, are thin bedded and softer. Where not fractured, it is a strong foundation rock and the practical problems concerning it have to do with the cracks which intersect it, and with the fact that it rests upon a weak formation.

The upper red shale is almost entirely devoid of the laminations typical of shale and, except for a few green strata, it appears unstratified. It consists, on the average, of 35% clay sizes (less than 0.005 mm), 45% silt sizes (0.005 to 0.05 mm), and 20% sand. Siltstone would be a more appropriate name than shale. The proportions of grain sizes vary considerably from place to place as do other characteristics of the shale. The percentage of clay sizes is greater in the south abutment than in the north abutment. The percentage of voids (that is, the ratio of the volume of voids to the total volume of the shale sample) varies both horizontally and vertically and is higher in the south abutment than in the north abutment. There is some tendency for the percentage of voids to be less in the lower part of the shale, but this characteristic is irregular in its distribution. The average percentage of voids in twenty-two samples of shale in the south abutment is 25.5; the maximum is 34.7, and the minimum, 12.6. The unit weight is 147 lb per cu ft in the natural condition. The average percentage of voids of forty-five samples from the north abutment is 17.6, the maximum being 26.0 and the minimum, 11.5. The specific gravity of the mineral grains averages 2.76. All the tests indicate that the shale is saturated with water. The red shale cracks upon exposure to air and crumbles to pieces. When re-wet, it breaks down to silt. If it is immersed in water in its natural condition without drying, however, it does not disintegrate. The most important foundation problems are concerned with this shale.

The pink shaly sandstone is an extremely variable formation ranging from a sandy shale to a fairly massive sandstone. It changes rapidly, both vertically and horizontally. Some parts disintegrate when dried and wet again, and other parts remain sound under such treatment. The practical problems are

due to this variability. The shaly parts of it present problems similar to those of a typical laminated shale.

The upper artesian sandstone is a hard, gray, fine-to-medium grained, gritty sandstone with occasional flat shaly seams; but it is generally massively bedded. Shaly beds are more numerous in the upper part of the formation where they were a foundation problem. It was given its name because water under artesian pressure was encountered in the borings. This fact necessitated studies concerning uplift and seepage.

The lower red shale is similar to the upper red shale, and the lower artesian sandstone is similar to the upper artesian sandstone. The total thickness of this sandstone is unknown because none of the borings reached the bottom of it. Both these formations are at such depth that they cause no practical problems.

South and southeast of Conchas Dam the lower part of the Dockum Group and certain other formations contain gypsum, but no beds of gypsum were found at Conchas Dam. It is possible, however, that gypsum underlies the dam at great depth. The deepest boring reached 240 ft beneath the river without encountering gypsum, and it is certain that if any exists beneath the dam it is so deep that it will be harmless.

The formations at Conchas Dam were laid down essentially flat in water and have been slightly tilted and warped so that they now have a regional dip of about 2° to the southeast upon which have been superimposed gentle waves or folds. One of these folds brings soft rocks to the surface in the valley of the Conchas River, with the result that this valley is wider than the canyon of the Canadian River. No evidence of faulting was found near the dam site or in the reservoir basin, and the nearest known fault is 34 miles west.

The rocks at the dam site are intersected by fractures or joints running in various directions. These cracks are most numerous in the canyon sandstone and artesian sandstone and least developed in the red shale. The joints are most numerous in the directions N 50° W and N 40° E, and they are in general nearly vertical. The joints show every gradation from incipient planes of weakness to open cracks. In the canyon sandstone near the canyon wall where blocks of sandstone could move toward the unsupported edge, some of the cracks are gaping fissures.

There are comparatively few joints in the red shale, but there are numerous other fractures in various directions, and dipping at various angles. The characteristic fracture of the red shale is a short, curved, striated or slickensided fracture running in any direction and dipping at any angle. These fractures are generally from 1 in. to a few inches in length with a maximum length of about a foot. Similar slickensided fractures were produced in the Soils Laboratory at Conchas Dam by shearing a specimen of the red shale under a heavy transverse load. These fractures are evidently caused by very small movements.

The formations at the dam site were formerly buried by younger formations to a depth of at least 1 200 ft. Remnants of these younger and higher formations can be seen in the near-by mesas. Between the mesas is a broad undu-

lating valley in which the river is entrenched in a canyon. Old river gravels give evidence that the river formerly flowed at the level of this old valley.

The dam crosses the canyon, rises above the canyon walls, and extends on to the cap rock on either side, as shown in Fig. 3. The central section of the high part of the dam is founded upon the upper artesian sandstone, but the two ends of the high section rest upon the pink shaly sandstone. The high section abuts against the canyon walls of red shale on either side of the canyon. The two ends of the dam which extend upon the cap rock will rest upon the canyon sandstone which overlies the red shale. From either end of the concrete dam, earth wing-dams extend to high land. The South Wing-Dam is on shale and sandstone of the variegated formation except for the northern end which rests upon the canyon sandstone. The North Wing-Dam is on the canyon sandstone, but the northern end will be against shale of the variegated formation.

The main part of the South Dike rests upon shale and sandstone of the variegated formation which provides a tight, satisfactory foundation for this structure. The southeasterly dip causes the top of the variegated formation to be lower at the south end of the South Dike, with the result that here the top is below the level of the crest of the dike and the south end of the dike rests upon sandstone of the townsite formation. The tunnel for the irrigation outlet is in shale of the variegated formation which is here overlain by a cap of townsite sandstone.

The Emergency Spillway rests upon the variegated shale and sandstone, but in places this is only a thin layer on the canyon sandstone and the foundation excavation was extended down to this sandstone. Where the spillway structure rests on shale the foundations were excavated to sufficient depth to remove the possibility of sliding of this low structure. These structures will not be discussed further since there are no interesting geological problems connected with them.

Practical Problems

The most important practical problems at the site of the main concrete structure were:

(*a*) The upper red shale as a foundation formation (its competency to support the design load, settlement of structures upon it, and the protection of the shale from disintegration);

(*b*) Seepage under the dam through the artesian sandstone, uplift pressure on the foundations of the dam from artesian water in this formation, and meeting these conditions by grouting and drainage; and the selection of foundation beds in this sandstone and in the overlying pink shaly sandstone, and the stability of structures resting upon them; and,

(*c*) Seepage under and around the dam through the canyon sandstone, and its prevention. Effect of the cracks in this formation on the stability of that part of the dam founded upon it.

Problems Connected with the Red Shale.—The red shale presented the most serious problems. It is an unlaminated siltstone which was deposited as a silty mud. Its consolidated condition is due to compaction and not to cementa-

tion. Following its deposition as mud other sediments were laid down upon it, and their weight compacted the mud, slowly squeezing the water out of the pores. As the compaction of the mud continued it became hard, silty clay and finally siltstone or shale. Erosion subsequently removed much of the load and the shale expanded somewhat but did not reach its original porosity. When load is again placed upon it there will be a renewal of compaction thus causing the settlement of any structure resting upon it. That part of the dam resting upon the shale will cause some compaction of the shale, with resultant slight settlement of the dam, even though the weight of the dam is much less than the weight of the overlying formations that have been removed.

Laboratory consolidation tests had been made on the shale from which estimates of settlement had been made, but there was some question as to the accuracy of these methods when applied to shale instead of to clay for which the tests had been designed; and it was desired to check the results.

From a consolidation test it is possible to estimate the load under which clay was originally consolidated.[4] Because of the close relationship between clay and uncemented shale it was possible to apply this method to this shale. In this case it was also possible to make an approximate estimate of the former load on the shale from field observations. Therefore, if these two estimates should agree there would be a check upon the laboratory method. The previous consolidation tests had been conducted with a maximum pressure of 25 tons per sq ft, and it was not possible to use higher pressures with the existing apparatus. It was evident that the original pressures were much higher and new apparatus was designed capable of testing the sample under a pressure greater than 100 tons per sq ft. The most reliable tests made with this apparatus indicated a former load of 64 to 70 tons per sq ft. Several miles south of the dam, Mesa Rica rises more than 1 200 ft above the river. The Ogallala Formation, the youngest known in this region, caps the highest part of the mesa. It is known that this high formation once extended unbroken to the Llano Estacado, about 15 miles to the south, and to the Canadian Escarpment 30 miles to the north. This high formation thus once overlay the dam site and it is possible to make an approximate estimate of the load that once rested upon the red shale. The difference of elevation between the summit of the mesa and the shale at the dam site is 1 250 ft, and this is the minimum over-burden on the red shale. Due to dip of the strata and other causes it may have been somewhat greater. Since the ground-water conditions at that time are unknown, the weight of this over-burden is not exactly known; but, using extreme assumptions gives values of 50 and 94 tons per sq ft for the two extreme conditions, with the most probable range between 60 and 70 tons per sq ft which checks with the laboratory estimate of 64 to 70 tons per sq ft and indicates that the laboratory tests are at least approximately correct.

Estimates of probable settlement of that part of the dam underlain by red shale were made by applying the laboratory results to the geological

[4] "The Structure of Clay and Its Importance in Foundation Engineering," by Arthur Casagrande, Assoc. M. Am. Soc. C. E., *Journal*, Boston Soc. C. E., April, 1932, Vol. 19, No. 4, pp. 176–179.

interpretation of the strata and gave a maximum settlement for extreme conditions of about 4 in. after a period of years. There will be differential settlement, therefore, between the parts of the dam underlain by the red shale and those parts resting directly upon the artesian sandstone. The condition being known, the dam was designed to meet them, and a special joint was devised between the different parts of the dam.[5] Differential movement between the monoliths underlain by the red shale and those resting upon the artesian sandstone can thus take place harmlessly.

The strength of the shale—that is, its competency to support the load it will be required to carry—was studied by geological observations in the field and the conclusions were checked by laboratory tests and by computations. In the north abutment the red shale is about 70 ft thick, the sandstone cap rock averages 20 ft thick, and the dam rises 95 ft above this. Therefore, the load on the shale is 20 ft of sandstone and 95 ft of dam.

The canyon walls were studied for miles above the dam and places were found where the sandstone overlying the shale is more than 100 ft thick. Thus, there is proof that the shale is sustaining a load as great as, or greater than, that of the dam, without failure. The canyon walls generally break down by weathering of the shale. The shale cracks and disintegrates upon exposure to air, the sandstone cap is undermined, and large blocks tip and slide into the canyon. The only places seen where the walls had collapsed due to failure of the shale were where the stream had undercut the cliff, producing stresses greater than any that can exist in the shale under the dam. The field observations thus indicated that the red shale is strong enough to support the dam. This conclusion was checked by laboratory tests.

A series of shear tests and of unconfined compression tests on samples of the shale were made in the Soils Laboratory at the dam, with the results shown in Table 1. Compression tests were made on 2-in. cubes. The results for

TABLE 1.—SHEARING RESISTANCE, SAMPLES OF RED SHALE FROM THE NORTH ABUTMENT

(All Units Are Pounds per Square Inch)

Elevation, in feet, above mean sea level	Shearing Stress, for the Following Normal Loads:					
	50	75	100	125	150	157
4 110	218	253	263	287	290	. . .
4 084	202	260	220	292	. . .	337
4 065	185	242	242	{310, 327}	. . .	. . .

samples from different elevations, in both the north and south abutments, are given in Table 2.

A study was made of the stresses in the red shale under the action of the completed dam. The following assumptions and constants were used:

[5] For description of joint, see "Dam Building on Difficult Rock," *Engineering News-Record*, June 9, 1938, pp. 808–809.

(*a*) The net section of the non-overflow dam, without heel or toe fillets, was used; the foundation was assumed level at Elevation 4 140, which is the top of the canyon sandstone; and the bottom of sandstone was taken at Elevation 4 120, and the bottom of red clay shale at Elevation 4 050;

(*b*) The foundation materials were considered as homogeneous and isotropic, acting in agreement with Hooke's law;

(*c*) Five different classes of assumptions as to the distribution of stress at Elevation 4 140, and the effect of the sandstone in distributing the load at Elevation 4 120, were studied (see Cases I, II, III(*a*), III(*b*), and IV);

(*d*) The modulus of elasticity of the concrete, sandstone, and shale was assumed equal in Cases I, II, and III(*b*); and the modulus of elasticity of the shale was assumed to be one-third of the modulus of the sandstone or concrete in Cases III(*a*) and IV;

TABLE 2.—COMPRESSIVE STRENGTH OF RED SHALE, IN POUNDS PER SQUARE INCH

Elevation, in feet, above mean sea level	SAMPLE NOS.										
	1	2	3	4	5	6	7	8	9	10	Average
(*a*) NORTH ABUTMENT											
4 110	660	725	675	1 600*	600	675	625	650	1 300*	610	652
4 084	975	865	400	1 025	875	675	825	865	565	1 040	811
4 065	340	300	385	325	350	325	500*	350	300	570*	334
(*b*) SOUTH ABUTMENT (PILASTER SHAFTS)											
4 100	438	363	400	413	350	525	375	400	....	....	408
4 078	388	350	550	375	375	700*	463	475	....	....	444

* Omitted from average.

(*e*) A cross-section of the dam and foundation was studied in each case, and no forces acting at right angles with the plane of the section were considered;

(*f*) Uplift and internal pore pressure were neglected;

(*g*) Unit weights were assumed at 62.5 lb per cu ft for water, and 150 lb per cu ft for concrete, sandstone, and shale; and,

(*h*) The reservoir was assumed permanently filled to Elevation 4 230 for the full reservoir condition.

The following possible assumptions as to stress distributions were considered:

Case I.—Straight line distribution of stress at Elevation 4 140 was assumed and the sandstone was considered effective in spreading the load. This case represents the minimum stress concentration that can exist in the shale foundation.

Case II.—Non-linear distribution of stress at Elevation 4 140 was assumed and the sandstone was considered effective in spreading the load. This case

gives stress concentration at the toe and heel in the plane of the base, but considerable distribution before the load is transmitted to the shale.

Case III(a).—Non-linear distribution of stress at Elevation 4 120 was assumed. The sandstone above Elevation 4 120 was assumed to act with the dam, but to be ineffective in spreading the load on to the shale.

Case III(b).—All assumptions were the same as in Case III(*a*), except for the substitution of linear distribution for non-linear at Elevation 4 120.

Case IV.—In this case, non-linear distribution of stress at Elevation 4 120 was assumed. The sandstone above Elevation 4 120 was assumed to act with the dam, and was considered to be effective in spreading the load at 45° with the horizontal.

Stresses were computed for all these cases. Case I is less severe than actual conditions; the assumptions in Cases II and IV may closely approximate actual conditions; but Case III(*a*) is the most severe condition possible. The results obtained are conservative and are the ones given herein. The maximum computed shear stress in the shale at Elevation 4 120 is 138 lb per sq in. The average shearing strength as computed from unconfined compression tests on twenty 2-in. cubes was 266 lb per sq in. This is for material unconfined, but at the point of maximum stress the shale will be confined by a monolith which will rise from the sandstone below the shale. The maximum stress for this condition will be about 35 lb per sq in., giving a factor of safety of about 7.6.

Laboratory tests and computations thus confirm the geological conclusions as to the competency of the shale to support the load of the dam. There are factors, however, which the laboratory tests and computations cannot take into consideration but which are included in the demonstration given by the canyon walls. The shale is intersected by cracks and planes of weakness along which the resistance to shearing is less. If only the laboratory results were considered there would remain the possibility that these fractures might so weaken the shale that it might fail under the increased load. The canyon walls, however, demonstrate that the shale is actually standing a similar or greater load without evidence of failure. Therefore, the field and laboratory methods together give greater assurance than either alone. The laboratory method gives definite values for certain cases and the field observations take into consideration factors that cannot be provided for in the laboratory. The conclusion must be, therefore, that the shale will not fail under the additional load to be imposed by the dam.

Another problem is caused by the cracking of the shale upon drying and its disintegration when wet again. Drying and disintegration of the shale proceed rapidly. Exposure of a few hours causes the formation of cracks, and every crack expedites drying by letting air into the mass of the shale. When the shale is again exposed to water it slakes and disintegrates rapidly to mud. As a result there are no natural exposures of shale, unless they have been formed very recently by the cutting of a stream, or otherwise. Where the red shale outcrops in the canyon wall the solid shale is concealed by many feet of cracked shale and chips.

Due to this rapid drying and disintegration of the shale it must be protected during excavation for foundations either by leaving several feet of shale in place until ready to pour concrete and then making the final excavation and placing concrete rapidly, or by painting the surface of the shale with an asphaltic or other protective preparation. The only places where this problem is important are at the two abutments where concrete is poured against vertical faces of shale, and the dam is stepped up from the lower sandstone to the canyon sandstone.

In order to avoid having any part of the dam rest directly upon the shale it was necessary to cut vertical faces in the shale which is 70 ft thick on the north side and 50 ft thick on the south side. On the south side the shale supports a load of cap rock 80 ft thick but on the north side the cap rock is only 20 ft thick. There was fear that on the south side a vertical face might not stand up under the heavy load of cap rock. To avoid the necessity of an unsupported face of shale at the south abutment, columns of reinforced concrete were constructed, extending from the sandstone below the shale to the canyon sandstone above the shale. These columns were designed to act as beams to give lateral support to the shale. Shafts were sunk in the shale under the edge of the cap rock, and concrete was poured in these shafts. Before the shale was excavated, a

FIG. 4.—SOUTH ABUTMENT OF MAIN DAM, SHOWING SUPPORT OF SHALE, JANUARY 29, 1938

monolith 54 ft away had been brought up to the level of the top of the shale and then the columns were braced against it. The shale was then excavated without difficulty, concrete panels were placed between the columns, and the monolith was poured[6] (see Fig. 4). This method was adopted before the

[6] For more complete description see "Dam Building on Difficult Rock," *Engineering News-Record*, June 9, 1938, p. 810.

difficulties at the north abutment were encountered. This procedure was not considered feasible at the north abutment on account of the thinness and broken character of the cap rock. Since the load on the shale was much less here, an attempt was made to excavate a vertical face without any support and to pour the concrete against the shale; but when the face was about 50 ft high pieces of shale began to spall off and it was not safe to proceed. Spalling continued until the shale face had broken back an average of a few feet, with a maximum of 7 ft as determined by actual measurements from a skip suspended from the cableway. (See Figs. 5, 6, and 7.) The cap rock did not collapse and was left overhanging. The face remained in that condition for three months, until it was cut back. Several plans were studied and one was adopted which not only removed all question as to the stability of the shale face but solved other difficulties as well.

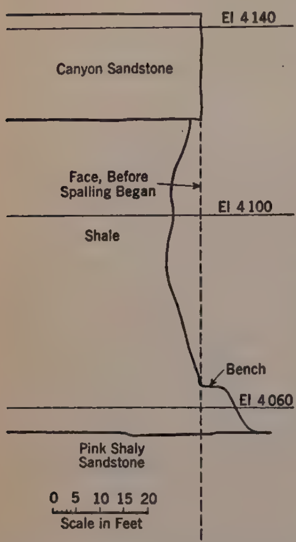

Fig. 5.—Section of North Abutment Breaking Back of Shale

A trench was excavated down through the red shale to the pink shaly sandstone and a monolith was poured in this trench. The trench was dug in sections and braced and the monolith was poured in sections. There was no difficulty with the shale during excavation of the trench. When the monolith was complete, the shale between it and the exposed face was excavated and the intervening monoliths were poured.

Practical Problems of the Artesian Sandstone and the Pink Shaly Sandstone

The artesian sandstone forms the foundations of the highest part of the dam and is a good rock for this purpose. It is moderately strong, its compressive strength as shown by eight tests ranging from 6 500 to more than 10 650 lb per sq in. There are problems, however, due to the fact that it is porous, is intersected by joint cracks, and contains water under artesian pressure.

The artesian sandstone was not exposed before excavations were begun, but it was investigated by numerous core borings and by two 30-in. borings, 76 and 81 ft deep, respectively, into which it was possible to descend and inspect the rock. Such large borings are the most satisfactory means of investigating foundation rocks. The smoothly cut surface of the holes shows up cracks, seams, and soft layers to much better advantage than the rough sides of shafts.

When the first holes were drilled into the artesian sandstone water flowed from them. The largest artesian flows were from the middle part of the upper artesian sandstone and from the lower artesian sandstone. The static head of water in the various holes was measured. The elevation of the river at the dam site is approximately 4 050 ft and the highest static head of water in the

Fig. 6.—North Abutment before Spalling of Shale, Showing Asphaltic Coating

Fig. 7.—North Abutment after Spalling of Shale, Showing Overhanging Sandstone Cap Rock

holes which penetrated the upper artesian sandstone only was 4 065 ft. The upper artesian sandstone outcrops in the South Canadian River two miles up stream from the dam at approximately the same elevation. The lower artesian sandstone outcrops farther up the river at a somewhat higher elevation and the highest static head of artesian water in holes penetrating the lower sandstone is 4 082 ft. Both these static heads agree with the elevations of the probable intake areas of the upper and lower artesian sandstones. Therefore, when the reservoir is full the water in these sandstones will be subject to the effect of this head.

The percentage of voids in the central part of the upper artesian sandstone varied from 6.4 to 22.7. The porosity of the upper 30 ft is comparatively low, and for the lower part, next to the lower red shale, it is also low. The permeability of the sandstone was tested both parallel and perpendicular to the bedding. The permeability parallel to the bedding is greater than it is perpendicular to the bedding, but even so it is low. The highest coefficient of permeability measured was 1.2 by 10^{-5} cm per sec. At this permeability water would flow through the sandstone under unit hydraulic gradient at the rate of 0.000024 ft per min. This is only 0.037 ft per day, an extremely slow flow, which could not account for a flow of 121 gal per min from some of the holes. This fact and others indicate that the artesian water moves principally through cracks rather than through the pores of the rock. Observations in the two 30-in. borings support this conclusion. Since the rock is porous and permeable, artesian pressure under static conditions will be equalized throughout it.

If the dam were built on this sandstone without a grout curtain and without drainage, the base would be subject to uplift pressure decreasing from the heel to the toe and the pressure would permeate the entire foundation rock. There might also be an undesirable seepage under the dam through the cracks in the sandstone. Seepage is reduced by a grout curtain extending down to the lower red shale, and uplift pressure is reduced by drainage. The conditions at this dam site are excellent for an effective grout curtain since the seepage will be principally through cracks, and the grout holes can reach an impervious layer at a moderate depth. Since seepage will be through cracks which are generally vertical, inclined holes are more effective in intersecting the greatest number of cracks.

The grout curtain consists of holes on 4-ft centers at the heel of the dam, dipping 75° S—that is, inclined 15° from the vertical. Every other hole extends down into the lower red shale and the alternate holes reach a vertical depth of 20 ft. Provision was made for closer grouting where necessary.

In rock with flat bedding seams, there is danger of opening the seams and lifting the rock by excessive grout pressure. Such uplift has been observed in some cases and since open bedding cracks were seen in the two large borings the possibility of this uplift was eliminated by restricting the pressure to the weight of the overlying material. Grouting was done after considerable concrete was poured and grout pressure was restricted to 1 lb per sq in. per ft of

overlying rock and concrete. In order to use higher pressures in the lower part of the holes they were grouted in stages using stops at a vertical depth of 20 ft.

The average grout consumption of 122 deep holes was 62.9 sacks, the minimum, 0.1 sack, and the maximum, 3 683 sacks. The average per foot of hole was 1.05 sack. Omitting the 2 holes that required more than 600 sacks each the average is 24.9 sacks per hole, or 0.42 sack per ft of hole. The average grout consumption of 111 shallow holes was 15.5 sacks, the minimum was 0.1 sack, and the maximum was 433.2 sacks. The average per foot of hole was 0.78 sack. Subsequent boring has produced cores with cracks thoroughly filled and two parts of the core firmly cemented together. In addition to the line of grout holes at the heel of the dam, provision is made for a line of holes from a gallery in the dam. These holes can be grouted at higher pressures after the dam is completed. At the abutments where the foundation is stepped up to the canyon sandstone the grout holes were inclined under the cliff and higher pressures were used.

Relief of uplift pressure is provided by a line of 8-in. drain holes from a gallery down stream from the grouting gallery. The average spacing of drain holes approximated 15 ft on centers. They were drilled to the red shale after all grouting had been completed. This program of grouting and drainage should solve the practical problems of the artesian sandstone. The drain holes will relieve uplift pressure and the grout curtain will prevent undesirable seepage.

Artesian aquifers similar to this sandstone are believed to be subject to compression under increased load and to expansion under increased artesian pressure.[7] Since the artesian sandstone will be exposed to reservoir water it will be subject to the fluctuating head of the reservoir and, theoretically, it should expand and contract. It is not known that any such movement has been recognized at any existing dam and it would probably affect a considerable area uniformly and be harmless. Further research on this point is desirable.

As has been mentioned there are lenticular shaly beds in the upper part of the upper artesian sandstone. These beds are not continuous and, therefore, the foundations of the different monoliths were excavated to different depths in order to reach beds of massive sandstone.

On either side of the river near the abutments it was not considered necessary to go down to the artesian sandstone and the foundations were placed on massive beds in the pink shaly sandstone formation. The only possible danger from this rock is in regard to sliding on some of the shaly bedding seams, but the excavations were carried down to such depth that sliding would involve shearing overlying beds of massive sandstone, which would be practically impossible. Unconfined compressive tests on the pink shaly sandstone showed a minimum strength of 2 240 lb per sq in. when wet, and a maximum of 6 940 when dry. Shear tests parallel to the bedding on typical samples in natural condition under a transverse load of 100 lb per sq in. gave a minimum strength of 441 lb per sq in., a maximum of 2 035 lb, and an average of 971 lb per sq in.

[7] "Compressibility and Elasticity of Artesian Aquifers," by Oscar Edward Meinzer, *Economic Geology*, Vol. 23, No. 3, 1928, pp. 263–291.

It is believed that the foundation beds selected in both the artesian sandstone and the pink shaly sandstone are thoroughly competent to support the loads imposed upon them.

Stability and Tightness of the Canyon Sandstone

The canyon sandstone at the north abutement was cut by many open cracks, and large blocks near the edge appeared to have moved. Therefore, questions were put to the geologist as to the cause of these cracks and their open condition, as to the cause of the movement of large blocks of sandstone, whether this movement would continue after the dam was built, and as to whether this rock was a satisfactory foundation for a dam 100 ft high. Furthermore, there was the problem of preventing seepage through cracks in the sandstone.

Most of the cracks in the sandstone are clearly joint cracks such as intersect the sandstones throughout this region. The two possible causes of opening of the cracks are solution and movement of the blocks. This sandstone is somewhat calcareous and in places shows signs of solution, but most of the cracks are free from such evidence. On the other hand it is obvious that many of the blocks near the canyon wall have moved, and since open cracks are restricted to the vicinity of the canyon, movement of the blocks is obviously the explanation. The cause of this movement is due partly to undermining by weathering and disintegration of the shale and partly to frost action and expansion from temperature changes. The blocks would move easily toward the canyon wall and there would be little tendency for them to move back.

The high part of the dam is extended back some distance from the canyon wall at the north abutment, resulting in the removal of the sandstone cap near the canyon with the consequent removal of all blocks that have moved. The problem, therefore, is to prevent future movement. When the dam is completed the outcrop of red shale in the canyon walls will be covered by an earth enbankment which will effectively prevent weathering of the shale near the dam and will thus prevent undermining of the sandstone. The earth embankment of the wing-dams will cover the surface of the sandstone to the canyon wall and will thus protect the sandstone from frost action and temperature changes. Therefore, the causes of movement of sandstone blocks will have been removed when the dam is completed, and there should be no future tendency for the blocks of the cap rock under or near the dam to move. Thus, by analysis of the causes of the unfavorable conditions, it was possible to remedy these conditions, prevent their future recurrence, and obtain assurance that the foundations will remain stable.

The canyon sandstone is a moderately strong rock, as is shown by the unconfined compression tests on six samples under different conditions, presented in Table 3. This formation varies from a hard massive sandstone to a softer thin-bedded sandstone, and the lower values were from samples of the softer sandstone.

From a foundation viewpoint the sandstone cannot be considered alone because it rests upon the red shale which is a much weaker rock. It has been shown that the red shale is strong enough to support the load of the dam but that there may be a total maximum settlement of 4 in. over a period of years. There may also be some adjustment due to plastic flow of the shale. The amount of settlement is not as important as its character (that is, whether it is irregular or even), and the way it is transmitted by the sandstone to the dam.

TABLE 3.—COMPRESSIVE STRENGTH OF CANYON SANDSTONE, IN POUNDS PER SQUARE INCH

Position, in respect to bedding	Strength of Sample Nos.:										
	1		2		3		4		5	8	
	Dry	Wet	Dry	Wet	Dry	Wet	Dry	Wet	Dry	Dry	Wet
Perpendicular	7 870	7 850	8 220	7 220	2 820	2 590	8 500	7 820	5 740	3 200	2 610
Parallel	8 550	7 030	4 130	4 070	2 310	2 150	7 670	6 410	8 120	6 020	3 280
45°	5 100	4 700	4 200	3 820	4 150	3 470	8 210	4 150	....	....	....

If the sandstone were a continuous, unbroken bed it would distribute any settlement effectively, but actually it consists of several beds intersected by numerous joint cracks. Since the cracks are offset from bed to bed, it may be likened to a mass of dry masonry with offset joints. It will thus be more effective in spreading the load than if the joints continued directly through all the beds, but less effective than an unbroken bed. The sandstone cap in the north abutment is only about 20 ft thick whereas at the south abutment it is about 80 ft thick, and, therefore, will spread the load much more effectively than at the north abutment. This was one reason for extending the high part of the dam farther back from the canyon wall at the north abutment. There will probably be some small uneven settlement of the different monoliths of the dam at the north abutment but provision has been made for it. The conclusion reached was that the canyon sandstone would provide satisfactory foundations for the dam.

There remains the problem of preventing seepage through the cracks of the sandstone. This is especially important because such seepage might tend to erode the top of the red shale. Consolidation grouting of the canyon sandstone to fill the cracks was required for the entire area of the concrete and earth dams; then a grout curtain was formed at the heel of the concrete dam by a line of inclined holes extending down to the red shale. This grouting was done after at least 25 ft of concrete had been placed, and grout pressures were restricted to 1 lb per sq in. per ft of overlying materials. Fifty-four shallow holes, each 20 ft deep, required 2.32 sacks of cement per foot, an average of 46.4 sacks per hole. The maximum was 935 sacks and the minimum was 0.0 sacks per hole. Twenty-six deep holes, averaging 7 ft, required 1.99 sacks per foot of hole or 77.4 sacks per hole. A comparison of inclined and vertical holes proves the greater effectiveness of the former. Thirteen deep inclined holes averaged 3.84 sacks per foot of hole, whereas ten vertical holes averaged

only 0.31 sack per foot of hole. The permeability of the sandstone is very low, the highest coefficient of permeability found being 2.8 by 10^{-5} cm per sec, or 0.000056 ft per min. This is comparable with the artesian sandstone, and seepage through the unfractured rock will be negligible. Since seepage will be through cracks and not through the pores, grouting should prevent seepage, effectively, through the canyon sandstone under the dam.

Conclusions

The foregoing description of Conchas Dam illustrates the problems of constructing a high concrete dam on a soft, weak shale, or siltstone. Such rocks are common in the Southwest and similar problems occur in other regions; but high concrete gravity dams on such foundations are rare. There are certain similarities between the foundation problems of Conchas Dam and Tygart Dam on the Tygart River, near Grafton, W. Va. The fire-clay shale, or indurated clay at Tygart Dam, is not laminated and is somewhat similar to the red shale at Conchas Dam; but it occurs in thinner beds separated by beds of strong sandstone, and the practical problems are less serious. The same is true of the State Line or Lake Lynn Dam, on the Cheat River, in West Virginia, which has been in use for more than ten years.

The investigation of the foundations of Conchas Dam has involved extensive geological field studies, not restricted to the immediate dam site, an extensive drilling program, large drill holes, tunnels and test shafts, physical, and chemical tests, and tests adapted from soil mechanics; but, nevertheless, it has been essentially a geological investigation, using every means available to solve geological problems. Where precise laboratory methods could be used it was necessary to apply their results to the conditions in the ground, and it was realized that conclusions from the precise tests could be no more accurate than the geological interpretation of conditions in the ground to which they were applied. The application of precise methods to geological data, which from their very nature cannot be expressed in terms of precise measurement, involves the exercise of judgment lest the seemingly precise solution by laboratory methods of parts of the problem may seem to indicate that the solution of the entire problem is precise. Such a conclusion would lead to unwarranted confidence and might result in disaster.

Since at practically no dam site is it possible to obtain all the desirable information, the importance of geological judgment and familiarity with similar sites becomes clear. Great care must be exercised, however, so that "judgment" will not be carried to the dangerous extreme of assuming that, because a site is similar to another site, a similar dam will result in similar satisfactory results. The only safe course, therefore, is to obtain all the information possible, use the best precise methods applicable, compare details with similar details elsewhere, use the best geological judgment, and then provide for the worst possible conditions. The last provision will take care of the impossibility of obtaining all desirable information and the use of judgment based on experience will obviate resort to unnecessarily expensive designs.

The geological investigation of Conchas Dam site has emphasized the need of additional research upon several problems. A better understanding is

desirable of the compaction of shale, and additional high-pressure "consolidation" tests on shales where the former load on them can be estimated should give valuable results.

Little is known about the slow plastic flow or creep of shale under pressures less than its shearing strength over a period of years, although there are many indications that such movement does take place. Laboratory tests, field tests on construction jobs, and systematic observations over a period of years on completed structures should give important information. Laboratory tests such as placing a specimen of shale under a constant load less than its shearing strength for months, and having provision for measuring shortening and lateral expansion of the specimen, should prove whether there is a slow deformation under moderate load. It is essential, however, to protect the specimen from drying during testing. If the elevation of a dam on uncemented shale were checked for a period of years against a datum well removed from the influence of the dam, and settlement of the dam were checked against the expected settlement as computed from consolidation tests, there would be an indication as to whether movement other than compaction were taking place in the shale.

It is known that artesian aquifers will expand when subject to artesian pressure and contract when that pressure is removed, but there is practically no information as to whether dams underlain by an artesian aquifer are subject to appreciable movement. Long-continued observations on dams, such as Conchas Dam, which rest upon an artesian aquifer that will be exposed to the fluctuating head of a flood control reservoir, should give valuable information. It would be necessary to refer the observations to a datum well down stream from the dam.

The problems of Conchas Dam are typical of those to be encountered with sites on the extensive red beds and similar formations. Since the most difficult problems—those concerned with settlement and plastic flow—may occur wherever uncemented shales occur, and since such formations are found throughout a vast area of the Southwestern, Western, and Northwestern plains, Conchas Dam illustrates some of the most important foundation problems of innumerable dam sites.

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PAPERS

THE RISK OF THE UNEXPECTED IN SUB-SURFACE CONSTRUCTION CONTRACTS

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Synopsis

In the early days before the development of a technique for preliminary underground exploration, it was customary for the contract to provide that the risk of the unexpected should fall on the contractor. Broadly worded contract clauses took the place of scientific and careful investigation. This practice often resulted in a higher total cost to the owner, since contractors, perforce, added an arbitrary sum to their bids to cover possible difficulties (1(*a*));[2] that is, the owner passed on the risk to the contractor and paid a substantial premium for this insurance against loss.

Modern standards of sub-surface investigation, as detailed in the Society's Manual of Engineering Practice No. 8 (1), have eliminated much of the costly uncertainty involved in underground construction, with corresponding benefit to owner and contractor. Indeed, the cost of supplying the contractor with complete data on subterranean conditions will generally be less than the sum contractors would charge for blindly assuming all risks.

Nature, however, often shows a whimsical unconcern for Man's data; a careful and complete preliminary investigation may not always forecast actual subsoil conditions accurately. In that event, the parties to the contract often disagree as to which must bear the loss. Even an agreement carefully drawn along the lines suggested in Manual No. 8 (1) may yet leave room for reasonable men to differ as to its interpretation; and, when amicable adjustment of the dispute is not effected, it is more than likely that recourse will be had to the Courts.

Experience in this type of litigation shows that emphasis is placed on carefully worded specifications, often with results expensively significant to one

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[2] For reference to numerals in parentheses, see "Court Citations and References," in the Appendix.

party or the other. A judicial ruling may result in an unexpected assessment against the owner or in wrecking the contractor.

This paper shows how reports of borings and similar data may return to plague both owner and contractor, even when given in the utmost good faith and with careful attention to the best modern practice. There should be a clear "meeting of the minds" in the perfect contract. It is necessary, therefore, that both parties to the contract understand the extent of the risks they are assuming, so that the hazards of litigation may not be added to the still substantial uncertainties of the subsoil.

Boring Data

In Manual No. 8 it is suggested that variations in subsoil conditions, revealed during the progress of the work, shall not entitle the contractor to additional compensation, unless the variance is considerable and the error in the preliminary boring data is firmly established (1(*b*)). However, extra compensation has been allowed by the Courts even without specific evidence that error was made in gathering the preliminary information.

In one case (2), the contractor had agreed to excavate a by-pass for Oklahoma City, Okla. Boring and sounding records appended to the contract led the contractor to believe that the material to be excavated consisted of a quantity of earth and dirt with a small quantity of rock. There was no language in the contract purporting to absolve the City from liability should the indications of the borings and soundings prove inaccurate. The contractor bid a unit price per cubic yard for "all excavation," which was to "cover the excavation of all earth of whatsoever nature down to the grades and lines as shown on the plans." Instead of the small quantity of rock indicated by the preliminary tests, about 60 000 cu yd were found, and suit was brought to recover the additional cost of excavating it.

The State Supreme Court of Oklahoma held that the contractor was entitled to recover, stressing the combined significance of two facts: (*a*) That the contractor was led to believe from the records of borings and soundings "that there was no rock to be encountered except in negligible quantities"; and (*b*) that the price bid, from the restrictive language of the specifications, merely covered the excavation of earth. The word, "earth," it was declared, did not include rock (3); therefore, the excavation of rock had not been "contemplated nor contracted for," and should be treated as extra work.

If, unlike the foregoing case, the language of the contract is broad enough to embrace removal of materials not shown by the borings (4), contract provisions designed to shift the risk to the contractor should the borings prove misleading are frequently effective, particularly in the absence of evidence that serious error was made in making the tests. Thus, it has been held that if the owner "does not warrant the indications of the borings to be correct," and if the contractor agrees that he shall have no claim "should the character and extent of the various materials be found to differ from what is indicated," owner responsibility for the unexpected is averted (5).

A similar ruling was made in a case in which the contract stated that there was no express or implied guaranty as to the accuracy of the borings and soundings, and directed each bidder to place his own interpretation upon the soundings and borings made by the City in forming his opinion of the character of the materials to be excavated (6).

The State of New York includes in its contracts a disclaimer clause which has proved an almost impenetrable barrier against claims based on unexpected sub-surface conditions. A leading case in point is Foundation Company *vs.* State of New York (7), where a recovery was denied in spite of the fact that the borings incorrectly indicated the elevation at which bed-rock would be encountered. The contractor there had agreed that he would make no claim against the State "because any of the estimates, tests, or representations of any kind affecting the work made by any officer or agent of the State, may prove to be in any respect erroneous." This disclaimer clause is ineffective, however, where fraud is proven (as discussed subsequently), and also where the extra expense incurred is due to a subsequent act of neglect or default on the part of the State (9).

Disclaimer clauses have even been held to cover a case in which the borings were interpreted incorrectly by the owner's engineer. The evidence in that case (10) was that the engineer's error was due to his ignorance of the wash-boring technique; the indications of hardpan were not recognized. The decision holding against the contractor's claim rested largely on a contract clause stating that the borings were exhibited to the contractor so that he could form his own judgment without implying any guaranty on the part of the owner as to their completeness or correctness. The Court held that, under this clause, an honest mistake would not justify a recovery unless it rose to "that degree sufficient to brand it as the equivalent of fraud or bad faith." A similar ruling was made in another case (11) where the contract stated that the borings were not guaranteed to be absolutely correct.

No disclaimer clause can force the contractor to assume the risk of deceptive borings, however, if the information given is known to be incomplete or misleading. The well established doctrine that no man may contract against his own "fraud" (12), strikes down any attempt on the part of the owner to escape liability for "unexpected" subsoil conditions. Although the principle is undeniably sound, difficulty lurks in determining just what facts will justify its application.

A well known case arose from a contract with the United States Government for the construction of three locks and dams on the Warrior River, in Alabama (13). Borings met "obstructions which, from the particles broken off and floating to the surface, would indicate that they might be logs." When this happened the drill was moved elsewhere until a place was found into which it would penetrate. The evidence of buried logs was not noted on the boring plans because the Government's engineer did not consider them of sufficient importance to be so recorded.

Although the Court found that this was an honest opinion and that there was no conscious intention to conceal material information from bidders, the contractor was permitted to recover the additional cost of uprooting stumps,

buried logs, cemented sand and gravel, and sand-stone conglomerate, none of which was indicated on the borings. The extent to which the Court's decision was influenced by the statement in the specifications that "the material to be excavated, as far as known, is shown by borings" is not apparent, but it is significant that, in its opinion, the Court italicized the words, "as far as known" (14). In any event, it would seem that the engineer's failure to note a significant detail on the boring plan may have an expensive sequel.

Even the broad disclaimer clause included in New York State contracts has been held impotent to prevent recovery by the contractor when material information has been knowingly concealed. In one case (15) the State had conducted earlier boring and sounding tests which suggested the presence of hardpan, but the results of these preliminary explorations were not disclosed to bidders. Instead, they were given a later series of wash-borings which did not show hardpan. Although the State's engineer testified that the earlier borings were not disclosed because, in his honest opinion, they were not as reliable an indication as those that were shown, the contractor was allowed to recover the additional cost of excavating hardpan. Recovery was also allowed in another case where the statute authorizing the construction of the New York State Barge Canal required the State to give bidders accurate information (16).

In still another case (17), it appeared that the Passaic Valley Sewerage Commissioners had contracted for the construction of a subterranean and submarine sewerage tunnel. A series of borings had been made and the results tabulated on a set of plans appended to the contract. These findings signified that under the waters of New York Bay the material to be excavated consisted of a layer of mud; below that a layer of sand and some boulders; and, beneath that, a "cemented Triassic formation."

On these data the Commissioners entered into a contract with a party which, after completing only part of the work and failing to find the "cemented Triassic formation," abandoned the job. The Commissioners then entered into a contract with a second party on exactly the same data previously used. Again, the elusive rock formation was not found; again, the contractor abandoned the work.

In re-advertising the contract for the third time the Commissioners, now apparently doubtful of the accuracy of the borings, added a note to the contract stating that the borings were "supposed to be approximately correct, but should they be found to be otherwise the contractor shall have no claim on that account, it being expressly understood that the Commissioners do not warrant the plot to be approximately correct." The contract was awarded, and for the third time the "cemented Triassic formation" was not found; instead, gravel, quicksand, bull-liver, boulders, and other pervious materials were encountered. This time the contractor stopped work and brought an action for damages. A verdict for the plaintiff was affirmed by the Circuit Court of Appeals. It was held that the self-exonerating note appended to the boring plans was ineffective to relieve the defendant from liability, inasmuch as the defendant knew from its two previous experiences that the borings were misleading (18).

In another case, it appeared that a plan of preliminary borings was included in a contract to construct a section of the Lexington Avenue Subway in the City of New York, N. Y. (19). The borings, although "not guaranteed," indicated that the tunnel was to be constructed entirely through solid rock, and the contract drawings were obviously based upon that supposition; but the drawings also contained a notation that "if rock is not encountered, the design of the side walls and floor will be modified." For a considerable part of the distance traversed the contractor met soft earth and mud that required a pneumatic method of tunneling, and to recover the higher cost of this type of construction an action was commenced. In allowing a recovery the Court emphasized the fact that on file in the office of the Public Service Commission there was a copy of the "Viele" map which showed that in earlier times a creek had crossed the line of work at the point where soft earth and mud were found. From this, the Court stated, the engineers for the Public Service Commission should have expected that soft earth or mud would there be encountered. It was concluded, therefore, that the extra work was caused by conditions which should have been anticipated by the Commission, and could not have been discovered by bidders (20). Testimony that the map was also filed in the New York Public Library and was readily available for inspection by prospective bidders was apparently not considered of any significance in this instance. The ready availability of the allegedly concealed information, however, has been emphasized in cases denying recovery (21). Greatly stressed was the bidder's inability to make his own borings, since the sub-surface lay under a principal thoroughfare of the city (22). By the same token, the bidder's opportunity to make borings has been given equal importance in cases denying recovery (23).

Although Manual No. 8 (1*c*) suggests that boring data be included with the contract and guaranteed as to correctness, it also advises that "soil conditions between borings may be assumed by the contractor at his own risk." The distinction thus drawn between guaranteeing merely the nature of the subsoil within the limited area encompassed by the boring pipe, and not the subsoil throughout the entire site, is one that is not readily accepted by a Court of law. Judicial interpretation of a boring plan as being merely a statement of what was found where the borings were made is rarely encountered (24).

Fig. 1(*a*) represents a situation where bidders are merely advised as to the results of the boring tests at the place where the borings were taken. Fig. 1(*b*), on the contrary, constitutes a statement not only of the results of the borings, but also of what is to be expected elsewhere.

A notable exception to the judicial reluctance to distinguish between these two situations is the opinion of the Circuit Court of Appeals in litigation over the construction of a bridge across the Arkansas River, at Fort Smith, Ark. (25). The contractor sought to cancel his contract because he found quicksand unexpectedly and because he did not encounter the soft shale and soapstone shown by the borings. The only statement in the contract pertaining to the borings was that "the findings are as indicated on sheet number 2," which, the Court said, amounted to nothing more than a statement that the findings of the borings were truly indicated. In its opinion the Court considered the inter-

pretation of borings in relation to construction contracts (26). It concluded that the borings were submitted merely as a bit of useful information, and not as a statement of sub-surface conditions generally to be expected throughout the site (27). There being no evidence that the results of the borings were not fully and fairly recorded on the boring plan, the Court held the defendant innocent of misrepresentation, and, accordingly, denied recovery.

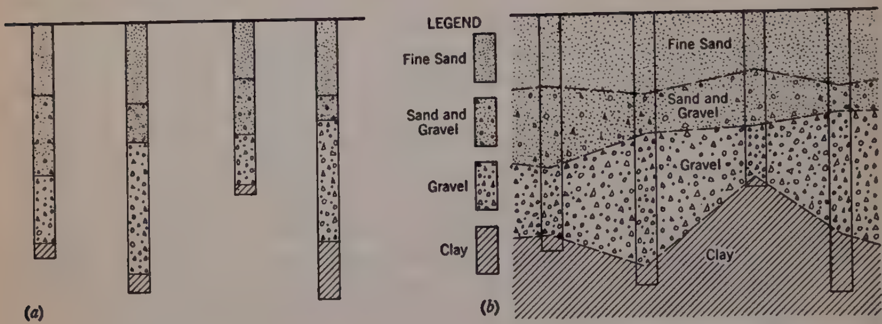

FIG. 1

This case eloquently illustrates to the two conflicting interpretations of a boring plan which Courts have been urged to adopt. Since the Court's understanding of borings is largely limited to what is testified before it by experts, irreconcilable decisions may be expected to continue as long as witnesses disagree.

OWNER INTERPRETATION OF BORINGS

In addition to a boring plan, the owner sometimes supplies his own conclusions as to what the borings show with reference to subsoil conditions throughout the entire site. Whether the owner thereby assumes any greater risk if his conclusion proves erroneous or misleading turns again on the Court's interpretation of the language used in the agreement. Thus, in a contract for the construction of a railroad bridge the contractor was directed to use a certain borrow-pit. The specifications stated that the borrow-pit was to be excavated to Elevation 655, except that no blue clay (which the borings indicated was approximately 15 ft below the surface) was to be taken (28). The contractor found the blue clay only 6.5 ft below the surface, and brought suit to recover the cost of obtaining additional fill elsewhere.

In the lower Court, the contractor won judgment on the theory that the railroad company had "falsely stated in the specifications that there was no blue clay in the tract nearer the surface than 15 ft"; but this judgment was reversed on appeal. The Appellate Court held that the lower Court had misconstrued the language used in the contract; the only statement made in the specifications was what the borings indicated, rather than what was the actual condition of the entire subsoil (29).

After constructing a lock and dam on the Ohio River up stream from Ashland, Ky., a contractor sued to recover the additional expense of removing

unexpected logs, snags, and stumps, found embedded in the river silt. These obstructions were not indicated on the boring plan, and the contract contained a disclaimer clause (30). The evidence disclosed that an earlier series of borings were not even shown to bidders and that one of these borings had encountered a "log next to rock." Nevertheless, a recovery was denied in view of the broadly worded disclaimer clause (31).

An interesting variation on this theme was presented in a Missouri case (32), in which the contract stated: "The borings indicate that all the tunnel will be in solid limestone." The borings did not justify this conclusion, however. The boring plan clearly indicated rock stratifications with seams of clay and gravel, which were plainly marked as "poor limestone"; and poor limestone was what the contractor found, not the "solid limestone" mentioned in the contract. Instead of breaking along the line of drilling, the rock broke off along the seams, necessitating additional expense for concrete reinforcement. Again, no recovery was allowed. In view of the fact that the borings, although not guaranteed, actually revealed the very condition later encountered, the contractor could not sustain the claim that he was misled by the owner's optimistic misinterpretation of them.

Profiles and Cross-Sections

Instead of a boring plan, there may sometimes be annexed to the contract a profile or cross-section showing the elevations of the surface and underlying rock, or the general composition of the subsoil throughout the site. In this case, the contractor is presented with the owner's conclusions as to sub-surface conditions, sometimes without access to the preliminary information upon which they were based. Must the contractor bear the loss if the true situation is found to differ from that indicated on the plan?

Here, again, the language of the contract is generally the decisive factor. In one case a profile plan showed the location of a proposed sewer in relation to the existing surface of the street (33). Superimposed on this plan was a convoluted, dotted, line marked: "Probable surface of rock as shown by borings." The contractor had agreed, however, that if "the location of the rock be found to differ from what is indicated," he "shall have no claim on that account." During excavation, quicksand was encountered where the profile showed rock, and the contractor sued for the added expense thereby occasioned. Recovery was denied. The Court held that the contract clause was a disclaimer of warranty relating "expressly to the convoluted line which the plaintiffs claim constitutes a warranty of the true location of the rock."

It has also been held that if the specifications state that "the profiles are reasonably correct, but are not guaranteed to be absolutely so," honest mistakes made in observing and recording the results of the borings upon which the profile plan had been based, were not actionable (11). Similarly, owner liability was not imposed when the actual physical conditions, although different from the profile, were "obvious to any careful observer." For example, where it was a matter of common knowledge that a saw-mill had formerly occupied the site of the work, the contractor in one case (34) could not recover the expense due to logs that were not indicated on the profile. Re-

covery was also denied in a case (35) in which the contract stated that "levels of the rock at various points are shown approximately on the plans, but the bidder must satisfy himself of the accuracy of these and of the rock surfaces in general."

On the other hand, the use of the word, "approximate," in connection with the information given, is not always effective to prevent recovery by the contractor. This was demonstrated in a Virginia case (36). Bidders on a proposed contract to construct a bridge over the James River, near Richmond, were furnished "a profile of existing conditions," which showed a line marked "rock" extending across the bottom of the river. The successful bidder discovered rock at a much lower elevation than that shown on the profile and sued to recover the cost of additional earth excavation. The City of Richmond argued in defense that the words, "approximate" and "plus or minus," before the elevation figures on the profile were sufficient to excuse it from liability for the error.

The Court rejected this argument and permitted the contractor to recover. It was said that the words, "plus or minus" and "approximate," as used on the profile, served to cover only "negligible deviations from entire accuracy"; they were intended to express the idea merely "that the line as shown upon the profile was nearly, but not exactly, correct." Since the actual conditions departed so substantially from those pictured on the profile, the qualifying words were held inadequate as a defense.

A contract for the construction of a new coagulation concrete basin in Alliance, Ohio, incorporated a plan showing, in cross-section, the location of the basin in relation to the existing grade, signified by a wavy, horizontal line marked "approximate original grade" (37). From the scale of the drawing, the contractor calculated that a general average excavation of about 9 ft and a fill of about 9 ft would be required. He found, however, that the actual average excavation was only about 3 ft, and brought suit for the cost of obtaining the necessary additional fill elsewhere.

A verdict for the defendant City was reversed on appeal and a new trial was ordered. As in the foregoing case (36), the Court held that the word, "approximate," would not protect the City in the face of so substantial a discrepancy. The attempt to charge the contractor with knowledge of the error, on the theory that the contour lines on another plan revealed the inaccuracy, was also unsuccessful. Proof that bidders do not customarily compute excavation from contour lines was accepted by the Court as sufficient to "excuse the contractor for not discovering the mistake."

Recovery was allowed in a similar case (38) where the piers were to be built "to the line and elevations given on the plans" and excavation was to be "done to the lines shown." Recovery was denied, however, where the specifications stated that the plans showed "approximate grades for excavation," but where the contractor was also directed to carry the excavation "to the foundation lines and grades as shown on the plan, or to such increased depth as may be necessary to obtain a firm foundation" (39).

In still another case the contractor complained because rock was found nearer the surface than suggested on the profile (40). His attempt to recover the cost of excavating additional rock was unsuccessful. Here, again, the line

on the profile was marked "approximate rock line" but the contractor had also agreed, under another clause, to bear "all losses resulting to him * * * because the nature of the land in or on which the work is done is different from what is assumed or was expected." The fact that the contractor had failed to protest upon discovery of the error was likewise deemed fatal to the claim (41).

The foregoing case (40) was later cited by the same Court when it refused to grant a contractor the cost of extra excavation required to reach bed-rock (42). This claim was based upon the fact that a cross-section showed the foundations of the proposed dam resting on bed-rock, which, from the scale of the plan, appeared to be all above Elevation 148. The plans were to be "considered as only approximate," and although it was necessary to go a considerable distance below Elevation 148 to reach bed-rock, no recovery was permitted.

In holding that this was a contract to rest the foundations on bed-rock wherever found, and not a contract to build "these piers with foundations at a certain depth" shown on the plans, the Court emphasized the fact that the contract included a contingent item of "200 cubic yards of caisson work below elevation 148." The inclusion of such an item was considered a clear indication that bed-rock might be found below Elevation 148. The following part of the Court's opinion has been quoted frequently (43):

"A contract and specification may contain representations as to existing physical conditions. If so, a bidder may rely upon them, even though it be provided that he shall satisfy himself by personal inspection and investigation as to their truth, where because of time or situation such investigation would be unavailing. (Faber *v.* City of New York, 222 N. Y. 255); or statements may be made on which the bidder, because of the language of the contract, cannot rely. He may have agreed that he will not. Then if they are made in good faith he takes the risk of their accuracy."

In litigation over the construction of an East River bridge, the same Court had earlier held that a plan indicating the existing rock surface and showing caissons sunk into it, was a warranty as to the true position of bed-rock (44). However, the specifications contained the added limitation that "bids are based upon depth of excavation indicated on the plans," and that "if excavation deeper than that shown on the plans is required it is to be paid for" at so much per cubic yard. Bed-rock was found from 8 ft to 9 ft higher than was shown on the plan. In order that the top of the caissons should not protrude over the water level and create an obstacle to navigation, the contractor was directed to excavate deeper into the rock.

The defense was based on a contract provision which required that "the contractor must assume the responsibilities for the difficulties encountered in sinking the foundations to bedrock or into it to whatever depth shall be determined upon." In view of the other contract provision requiring payment for additional excavation into rock, the Court decided that the argued clause contemplated the assumption of responsibility merely for any "difficulties" encountered in excavating into rock, but not for the additional "expense" thereby occasioned!

Sub-Surface Described in Contract

A statement describing the materials to be encountered may be incorporated in the contract. The form of the alleged representation in this case is the language of the contract, and not mere lines or symbols on a plan whose meaning may be doubtful. Typical is the well known case (45) based on a United States Government contract for the repair of a dam, in which it was stated definitely that: "The dam is now backed for about fifty feet with broken stone, sawdust and sediment * * *." Instead, the contractor found a log cribwork filled with stones, and sued to recover the consequent additional expense. The contract provision requiring bidders to examine the site and to satisfy themselves as to the nature of the work to be done was urged in defense. With this argument the Court was unimpressed. A recovery was permitted because "the specifications spoke with certainty as to a part of the conditions to be encountered by the claimants," and of them, it was said, "the government might be presumed to speak with knowledge and authority."

In another case (46), recovery was permitted because the specifications stated falsely that "the vitrified pipe line is mostly, and the tunnels are entirely, to be in soft shale rock."

A similar conclusion was reached in a case arising out of a contract to excavate a ship channel at the mouth of the Detroit River (47). The material to be removed was stated to consist of: "Sand, gravel and boulders, all in unknown quantities"; but a large part of the material proved to be limestone rock. For the consequent additional cost of performance, the contractor was allowed reimbursement.

A contrary decision was given in an instance in which the statement as to the nature of the sub-surface was qualified (48). The contract stated that: "Good gravel will be encountered in the excavation," but the contractor was also notified that if sufficient gravel was not found, equally good gravel would have to be furnished for making concrete. The contractor, not finding any gravel at all, sued for the cost of obtaining it elsewhere. The Court held that no representation had been made to the contractor that he would find sufficient gravel in the excavation for making concrete; on the contrary: "All that the specifications say is that some gravel will be found in the excavation. The language of the specifications put him upon notice and required him to make investigation, and having failed to do so he cannot be heard to complain."

Recovery was likewise denied where the contract stated that a supply of necessary stone "could be obtained at Union Spring [N. Y.], at 90 cents the cubic yard" (49). Before bidding, the contractor made no attempt to verify this statement and, later, when he discovered that the stone could not be obtained at Union Spring, he tried to rescind the contract and recover his deposit. The broad disclaimer clause found in contracts with the State of New York, to which reference has been made heretofore, was cited by the Court in support of its conclusion that the contractor was not entitled to rely upon this statement as to source of supply.

In a very similar case, however, the clause was held impotent as a defense to the contractor's claim for the additional cost of obtaining stone at a place

other than that mentioned in the specifications (50). Before bidding, the contractor had inquired at the place named in the specifications and was told that the stone was available. Later, when he sought to purchase the material, he discovered that the person in possession had no legal right to sell it, and thus he was forced at added expense to go elsewhere.

Having done everything possible to verify the statement and having found that, apparently, it was true, the contractor was deemed entitled to rely upon it; whereas, in the preceding case (49), recovery was denied because investigation by the bidder would have disclosed the error. Thus, in the face of contract disclaimer clauses, investigation seems to be the wisest course.

As has already been shown in other types of cases, however, exonerating language in a contract is unavailing if the representations made are knowingly false. This was the situation presented in a contract which stated that "the material to be removed is believed to be mainly mud, or mud with an admixture of fine sand, except from Station 54 to Station 55 plus 144 * * * where the material is firm sand and gravel or cobbles" (51). The "accuracy of this description" was expressly not guaranteed and bidders were enjoined to decide for themselves the character of the materials to be handled. The description presumably was based on preliminary borings made by the Federal Government, which also were included in the contract with the statement that:

"No guarantee is given as to the correctness of these borings in representing the character of the bottom over the entire vicinity in which they were taken, although the general information given thereby is believed to be trustworthy."

Upon finding a heavy compacted material instead of mud and fine sand, the contractor discontinued operations and sued to recover the value of the work already done. It was shown that the Government had encountered an impenetrable material during preliminary borings and had not noted this fact on the plans. Recovery was allowed by the United States Supreme Court. The decision apparently turned upon the fact that the Government's assertion of "belief" as to the nature of the material was knowingly false, in that it was contrary to other information in its possession. (Compare this statement with that in Pearson *vs.* State of New York (8) in which the use of the word, "expected," was considered to be merely a statement of "hope or expectation" and not a "positive agreement or warranty.") Emphasized, too, was the fact that before work was begun, Government engineers had inspected the contractor's plant and equipment and approved it, knowing that it was suitable for dredging only the materials stated in the specifications.

Frequently, the statement or representation is not made part of the written contract. When such information proved incorrect, recovery was denied in an early case (52). The Court declared that a statement of the expected nature of the sub-surface was a mere expression of opinion "with reference to matters equally within the power of both of the parties to the contract to ascertain and determine for themselves," and that the failure to include this statement in the written contract indicated that the parties did not regard it as material.

Implied Subsoil Representations

It has occasionally been claimed that a plan, or the quantities stated in an engineer's estimate, or the methods prescribed for doing the work, led the contractor to believe that a certain type of sub-surface would be encountered. Such attempts to "spell out" an implied warranty have rarely been successful.

Plans.—In litigation over a contract to construct a sewer, it appeared that a plan showing the work to be done bore certain dotted lines which the testimony showed "was the usual manner of indicating an existing sewer" (53). The contractor claimed this authorized him to assume that there was such a sewer and that it would drain off all water flowing into or accumulating in the excavation during performance. In fact, no such sewer existed. An action was brought to recover the cost of keeping the trenches dry with pumps. Recovery was denied because of contract provisions requiring the contractor to keep all the trenches free from water while the excavation or the construction of the foundations and sewer was in progress. This provision negatived the idea that an outlet for water was to be supplied to the contractor by any existing sewer (54).

This ruling was repeated in an almost identical case (55), in which the contract also contained a similar clause requiring the contractor to keep the excavation free from water. In its opinion, the Court observed that, even if there had been a warranty that a sewer existed, there was certainly no warranty that it would be "fit and suitable for drainage."

In 1899 the City of New York entered into a contract to demolish the old reservoir at Forty-Second Street and Fifth Avenue to clear the way for the construction of the New York Public Library (56). Annexed to the contract was a plan which indicated the dimensions of the old reservoir walls, the angle of the slope, and the extent of the excavation and fill to be made. It did not, however, "show the quantity of material to be removed in removing the walls and foundation of the reservoir, or the quantity of this material that might be utilized upon the site for the filling required by the contract." The contractor asserted that these quantities could be calculated from the plans on the assumption they were correct and drawn to scale. The plans were inaccurate in that they indicated an angle of slope of 1 on 1 when actually it was 1 on 1.51. Consequently, about 20 000 more yards of material had to be removed than the contractor had calculated. Recovery for the extra work was denied. Emphasis was placed upon the fact that the contractor had agreed in the contract to perform "the entire removal and disposal of the masonry and rubbish of the reservoir" (57).

In a contract for the construction of a part of the Catskill Aqueduct a plan showed the types of tunnels to be constructed (58). Beneath the invert and resting on the bottom of the excavation, square and round drains were shown. These drains were to be installed so that "the bottom of the tunnel may be free from water and sufficiently dry at all times." Dimensions were given for all details on the plans except the drains, but their size could be determined by scaling. The contractor contended that a drain of this size could not carry off more than 70 to 80 gal per min. It was argued, therefore, that the drains shown

on the plans implied a warranty that the water to be handled would not exceed that quantity. Finding it necessary to dispose of as much as 560 gal per min, the contractor sued to recover the extra cost of drainage and of excavating rock under these unusually wet conditions.

Recovery was denied. The Court stressed contract provisions which called attention to the uncertainty of the sub-surface to be encountered and which directed the contract or to provide all necessary pumps, pipes, and drains at his own expense (59).

An ingenious argument was advanced for the implication of a warranty in a case in New York City (60). The contractor was directed to remove "earth, rock or other material overlying the sub-grade" of a roadway. When rock was found, he objected that its removal was not required by the contract. He pointed out that, many years before, the City had let a contract for the removal of all rock "overlying the sub-grade," and that the work done under that contract had been accepted by the City. He claimed, therefore, that he was justified in assuming that the other contractor had removed all the rock, and that there would be none left for him. His suit to recover the cost of removing the rock was unsuccessful. The Court held "that the acceptance by the City of the prior work was not a representation of anything, upon which this contractor was entitled to rely" (61). Of interest in this connection is a case decided by the New York Court of Appeals in 1938, involving the construction of the New York State Building in the City of New York, wherein the superstructure contractor was held entitled to recover damages for the delay occasioned by the fact that the foundation plans had to be altered to meet unexpected subsoil conditions (62). The foundation plans, it was said, were a warranty to the superstructure contractor as to the anticipated time for the commencement of his work.

Engineers' Estimates.—Attempts to base claims for damages upon an engineer's inaccurate estimate of the quantity of work to be done have been generally unsuccessful. The usual contract clause stating that such estimates are "approximate only, * * * are given to form a basis of comparison for testing bids, * * * are not guaranteed to be accurate, and are not considered as a binding feature of the contract," is generally sufficient to negative the existence of the warranty.

In several cases (63) where recovery for work in excess of the quantities included in the engineers' estimate was denied, the rulings apparently were centered around the proposition that the estimates formed no part of the contract and were ineffective to limit the amount of work to be done thereunder. As a corollary to this proposition, it has frequently been held that no action can be maintained for loss of profits if the engineer has over-estimated the quantity of work to be done (64).

Implications in Specified Methods.—Closely related to the principle that an owner impliedly warrants the sufficiency of his plans and specifications to accomplish the intended result (65), was the situation presented in litigation over the construction of a sewer in Yonkers, N. Y. (66). The contractor argued that the contract contained an implied warranty that the sewer tunnel was to be constructed in "free air." The claim was based on the fact that the contract

"set forth various features of the construction work required of the contractor which could be used only in a free air tunnel." Those "features" were: (1) The installation of under-drains; (2) the drilling of alignment holes; and (3) ventilation, concrete work, and care of water to be done by methods suitable only in a "free-air" job.

For 6 142 ft of 8 553 ft of tunnel, the contractor encountered a soft, water-bearing soil which required construction by compressed air. Suit was brought to recover the higher cost of doing the work in this manner.

The Court held that "the stipulation that these free air methods be used was a representation that the job was free air tunnel construction work," and that the contractor was entitled to recover. Of the contract provisions urged in defense (67), the Court stated that they could not be assumed to nullify the evidence in the plans that the contract was primarily for a free air tunnel (68); but the Court also observed that the contractor would not have been entitled to rely upon the representations if the borings, annexed to the contract, had indicated the presence of the water-soaked material. The attempt to advance this argument in defense failed because the testimony did not establish clearly what the borings really indicated.

In another case, the contractor argued that a warranty as to the character of the rock to be removed was to be inferred from the fact that the specifications provided for the use of light charges of explosives and for the completion of the entire job within six months (69); but the contention was unsuccessful. Recovery was denied. The Court held that no warranty could be implied, "particularly in view of the fact that there is no evidence indicating that by the use of more men and the employment of more equipment the task could not have been completed on time."

In litigation resulting from a contract for the construction of three giant piers in New York Harbor, the contractor argued, among other things, that a warranty as to the nature of the rock to be removed was to be inferred from the fact that vertical faces of the rock walls were to be produced by "channeling, or by drilling and broaching" (70). It was argued that a vertical face could not be produced by the methods specified unless the rock was firm and solid, and that, therefore, the contractor was justified in assuming that the rock would be of that type. The attempt to charge the City of New York for the extra expense of removing a large quantity of loose, rotten, shattered, and disintegrated rock failed. The evidence indicated that the specified methods would have been prescribed even for the kind of rock actually encountered. Therefore, the provision for "channeling or drilling and broaching" meant nothing at all with reference to character of the rock to be removed.

In another case, a contract provision that "tunneling will not be allowed without the approval of the engineer, and the method of tunneling shall be subject to his approval," was successfully used in defense of a claim for extra payment for the cost of removing an unexpectedly large quantity of rock (71). It was held that the contract clause should have warned the contractor that rock might be encountered and, accordingly, cast upon him the burden "of ascertaining for himself just the conditions to be encountered in the building of the drain or sewer."

Of course, if the contractor is left free to develop his own methods of accomplishing the result intended and is not circumscribed by specific contract requirements (72), the argument for the implication of this type of warranty cannot be made.

Conclusion

In the last analysis, each case, both past and future, must rest upon its own distinctive facts (73); but, in this matter, as in everything, a knowledge of what the Courts have done in similar circumstances lessens the element of uncertainty for the future. Even if it is too much to hope that risk can be eliminated entirely from sub-surface construction contracts, its existence and extent can be recognized in advance with the aid of earlier judicial decisions, to the end that costly mistakes for owner and contractor may be minimized.

APPENDIX

Court Citations and References

(1) "Engineering and Contracting Procedure for Foundations": Manual of Engineering Practice No. 8, Am. Soc. C. E.: (*a*) p. 15, "Compensation for Unforeseen Conditions"; (*b*) p. 4, "Contractor's Responsibility"; and (*c*) p. 12, "Borings, or Test Pits."

(2) Maney *vs.* Oklahoma City, 150 Okla. 77, 300 Pac. 642 (1931).

(3) To the same effect, see Sweeney *vs.* Jackson County, 93 Oregon 96, 178 Pac. 365, 376 (1919); Nesbett *vs.* Louisville, C. & C. R. R. Co., 2 Speers (S. C.) 697, 705 (1844); and Dickinson *vs.* City of Poughkeepsie, 75 N. Y. 65, 76 (1878).

(4) Where a contract provision is inconsistent with the plans, the former usually prevails. See Dean *vs.* Mayor, 167 N. Y. 13, 60 N. E. 236 (1901); Lentilhon *vs.* City of New York, 102 App. Div. 557, 92 N. Y. S. 897 (1905), affirmed 185 N. Y. 549, 77 N. E. 1190; Cunningham *vs.* City of New York, 39 Misc. 200, 79 N. Y. S. 401 (1902), affirmed 90 App. Div. 606, 85 N. Y. S. 1129; Barash *vs.* Board of Education, 226 App. Div. 249, 235, N. Y. S. 30 (1929), affirmed 255 N. Y. 587, 175 N. E. 324.

(5) O'Brien *vs.* City of New York, 15 N. Y. S. 520, affirmed 65 Hun 112, 19 N. Y. S. 793 (1892), affirmed 139 N. Y. 543, 35 N. E. 323, reargument denied 142 N. Y. 671, 37 N. E. 465.

(6) Odell *vs.* City of New York, 206 App. Div. 68, 200 N. Y. S. 705 (1923), affirmed 238 N. Y. 623, 144 N. E. 917. The contract clause provided as follows: "Most of the borings were made by means of wash drills, but many were made with core drills of various types. The test pits have been left open and may be inspected. There is, however, no expressed or implied guaranty as to the accuracy of the borings and soundings, nor of any interpretation of them. Each bidder must form his own opinion of the character of the materials to be excavated, from an inspection of the ground, put his own interpretation upon the soundings and borings made by the city, and make such other investigations as he may see fit."

(7) 233 N. Y. 177, 135 N. E. 236 (1922).
(8) Pearson & Son *vs.* State of New York, 112 Misc. 29, 182 N. Y. S. 481 (1920); Murray *vs.* State of New York, 131 Misc. 262, 226 N. Y. S. 625 (1928).
(9) Cunningham, Woodward Company *vs.* State of New York, 119 Misc. 761, 197 N. Y. S. 343 (1922); Sundstrom *vs.* State of New York, 213 N. Y. 68, 106 N. E. 924 (1914); Feeney & Sheehan Building Co. *vs.* State of New York, 120 Misc. 240, 198 N. Y. S. 83 (1923).
(10) The Arthur A. Johnson Corporation *vs.* City of New York, 95 N. Y. L. J. 3049 (1936); affirmed 251 App. Div. 811, permission to appeal denied, 276 N. Y. 688.
(11) City of Lima, Ohio *vs.* Farley, 7 Fed. (2d) 40 (C. C. A. Ohio, 1925).
(12) Bridger *vs.* Goldsmith, 143 N. Y. 424, 38 N. E. 458 (1894); McGovern *vs.* City of New York, 202 App. Div. 317, 195 N. Y. S. 925; affirmed in 235 N. Y. 275, 139 N. E. 266 (1925); Brassil *vs.* Maryland Casualty Co., 210 N. Y. 235, 104 N. E. 622 (1914) L. R. A. 1915A, 629.
(13) Christie *vs.* United States, 237 U. S. 234, 35 Sup. Ct. 565 (1914).
(14) 237 U. S., at p. 239. See also, comment on the case in Pearson & Son *vs.* State of New York, Note 8, and in Elkan *vs.* Sebastian Bridge District, Note 25.
(15) Jackson *vs.* State of New York, 210 App. Div. 115, 205 N. Y. S. 658 (1924); affirmed in 241 N. Y. 563, 150 N. E. 556 (1925).
(16) Stewart & Co., Inc. *vs.* State of New York, 121 Misc. 827, 201 N. Y. S. 334 (1923); affirmed in 218 App. Div. 810, 245 N. Y. S. 638.
(17) Passaic Valley Sewerage Comm'rs. *vs.* Holbrook, Cabot & Rollins Corporation, 6 Fed. (2d) 721 (C. C. A., 1925).
(18) The Court said: "The representations were false in the sense of being different from the conditions which, from the borings and the disclosures made in the O'Rourke and Haskins operations, the Commissioners knew to exist." (6 Fed. (2d) at p. 723.)
(19) McGovern *vs.* City of New York, 202 App. Div. 317, 195 N. Y. S. 925; affirmed in 235 N. Y. 275, 139 N. E. 266 (1925).
(20) 202 App. Div., at pp. 332–333; 195 N. Y. S., at p. 937.
(21) See Rodgers & Hagarty, Inc., *vs.* City of New York (1936), 247 App. Div. 874 (Record on Appeal, Folio 3951–3955); Irving Trust Company, as Trustee *vs.* City of New York (1937), 253 App. Div. 714 (Record on Appeal, Folio 1176–1178); Allen N. Spooner & Son Inc. *vs.* City of New York, 97 N. Y. L. J. 1439 (1937).
(22) See also Christie *vs.* U. S., Note 13; Jackson *vs.* State of New York, Note 15; Faber *vs.* City of New York, 222 N. Y. 255, 118 N. E. 609 (1908).
(23) Dunn *vs.* City of New York, 205 N. Y. 342, 98 N. E. 495 (1912); Leary *vs.* City of Watervliet, 222 N. Y. 337, 118 N. E. 849 (1918); Semper *vs.* Duffy, 227 N. Y. 151, 124 N. E. 743 (1919).
(24) Groton Bridge & Mfg. Co. *vs.* Alabama & V. Ry. Co., 80 Miss. 162, 31 So. 739 (1902); Pope *vs.* United States, 76 Ct. Cl. (U. S.) 64 (1932).
(25) Elkan *vs.* Sebastian Bridge District, 291 Fed. 532 (1923). The Court said: "Of course any one would realize that the actual subsoil conditions

might, except where and to the depth shown by the borings, be different than so shown. The actual conditions were hidden. The borings were merely indications, at certain places and to certain depths, from which deductions might be drawn as to actual conditions along the line and to the depths of such borings. Both parties knew that deductions so drawn might prove untrue when the necessary excavations were made."

(26) The Court said: "The question here is who took the risk of these deductions? Either the district or the contractor must take this risk. Either could legally do so. Construction of the contract in the light of applicable legal principles must determine which did. That the district made known to Elkan the result of the borings is certain! This was done with one of two purposes: (1) To be the statement of a fact (subsoil conditions to be encountered) upon which he might rely and act; or (2) as useful information which he might accept as sufficient or not as he thought wise, leaving him to make such further and other investigations as he might wish." (291 Fed., at p. 538.)

(27) The Court said: "The bare statement that the boring sheet may be relied upon as accurate is entirely different from saying that the sub-soil along the bridge line is as shown by the boring sheet. One is a statement that certain evidence of an ultimate fact exists; the other is a statement that the ultimate fact exists. The statement here made goes, in effect, little further than if the boring sheet had, without comment, been furnished the bidder." (291 Fed., at p. 540.)

(28) The exact wording was as follows: "The excavation of this borrow-pit shall be carried to Elevation 655 D. and I. Datum, or to blue clay, which test borings show to be approximately fifteen (15′) feet below the surface. If the blue clay is above Elevation 655, none of the blue clay shall be excavated or placed in the embankment." (Detroit & I. Ry. Co. *vs.* Guthrie & Co., Inc., 72 Fed. (2d) 126 (C. C. A. 1934).)

(29) The Court wrote: "The only representation of fact made therein was that test borings had been made, and that they showed blue clay approximately 15 feet below the surface. The issue to be submitted to the jury was not whether there was blue clay nearer the surface than fifteen feet, but whether the representation that borings had been made and they showed no blue clay less than fifteen feet below the surface was true or untrue." (72 Fed. (2d), at p. 129.)

(30) The contract provided: "From borings and test pits, made at the site, it is assumed that rock will be found approximately as indicated on the drawings, but the United States does not guarantee the nature of materials to be encountered in the river bed, the correctness of the depth of rock as assumed or shown on the drawings, the depth to which it may be necessary to excavate, nor that the bottom of the river will not be changed before or after commencement of work." (Bates & Rogers Construction Co. *vs.* United States, 56 Ct. Cl. (U. S.) 49 (1921).)

(31) The Court said: "It would be extending the rule further than the adjudicated cases have gone, and further than we are prepared to go, to say that the Government can be held, as upon a warranty, of a condition as to which it makes no representation * * * when all it does is to furnish drawings which accurately state all they purport to show, notwithstanding it fails to state to the bidder that a hole was bored somewhere in or about the large area involved at some other time which disclosed that there was a log at the bottom of the Ohio River." (Bates & Rogers Construction Co. *vs.* United States, 56 Ct. Cl. (U. S.), at p. 60.)

(32) United Construction Co. *vs.* St. Louis, 334 Mo. 1006, 69 S. W. (2d) 639 (1934).

(33) Kelly *vs.* City of New York, 87 App. Div. 299, 84 N. Y. S. 349 (1903); affirmed 180 N. Y. 507, 72 N. E. 1144.
(34) Hill *vs.* City of Beaumont, Texas, 5 S. W. (2d) 590 (1928); cited in dissenting opinion in City of Dallas, Texas *vs.* Shortall, 87 S. W. (2d) 844 (1935). See, also, Jahn Contracting Co. *vs.* City of Seattle, 100 Wash. 166, 170 Pac. 549 (1918).
(35) Conners *vs.* U. S. 130 Fed. 609 (1904); affirmed in 141 Fed. 16 (1905); see, also, Pawling *vs.* United States, 62 Ct. Cl. (U. S.) 123 (1926) and Cauldwell-Wingate Company *vs.* State of New York, 276 N. Y. 365 (1938).
(36) City of Richmond *vs.* I. J. Smith & Co., 119 Va. 198, 89 S. E. 123 (1916).
(37) Pitt. Const. Co. *vs.* City of Alliance, Ohio, 12 Fed. (2d) 28 (C. C. A. 1926).
(38) United Construction Co. *vs.* Haverhill, 9 Fed. (2d) 538 (C. C. A. 1925); see, also, same case in 22 Fed. (2d) 256 (C. C. A. 1927).
(39) Maryland Casualty Co. *vs.* Board of Water Commissioners, 43 Fed. (2d) 418 (Dist. Ct. 1930).
(40) Leary *vs.* City of Watervliet, 222 N. Y. 337, 118 N. E. 849 (1918).
(41) To the same effect see Borough Construction Co. *vs.* City of New York, 200 N. Y. 149, 93 N. E. 480 (1910); Simpson *vs.* United States, 172 U. S. 372, 19 Sup. Ct. 222 (1899); Marshall Construction Co., Inc. *vs.* State of New York, 133 Misc. 131, 231 N. Y. S. 345 (1928).
(42) Foundation Company *vs.* State of New York, 233 N. Y. 177, 135 N. E. 236 (1922).
(43) 233 N. Y., at pp. 184, 185; 135 N. E., at p. 238.
(44) Faber *vs.* City of New York, 222 N. Y. 255, 118 N. E. 609 (1918).
(45) Hollerbach *vs.* United States, 233 U. S. 165, 34 Sup. Ct. 553 (1913).
(46) Delafield *vs.* Village of Westfield, 77 Hun (N. Y.) 124, 28 N. Y. S. 440 (1894).
(47) United States *vs.* Smith, 256 U. S. 11, 41 Sup. Ct. 413 (1921).
(48) Kuhs *vs.* Flower City Tissue Mills Co., 104 Misc. 243, 171 N. Y. S. 688 (1918); modified in 189 App. Div. 539, 179 N. Y. S. 450.
(49) Matter of Semper *vs.* Duffy, 227 N. Y. 151, 124 N. E. 743 (1919); see also, Gross & Son *vs.* State of New York, 214 App. Div. 386, 212 N. Y. S. 222 (1925); Brennan Construction Company *vs.* State of New York, 117 Misc. 816, 191 N. Y. S. 253 (1921).
(50) Atlanta Construction Co. *vs.* State of New York, 103 Misc. 233, 175 N. Y. S. 453 (1918).
(51) United States *vs.* Atlantic Dredging Co., 253 U. S. 1, 40 Sup. Ct. 423 (1930).
(52) Nounnan *vs.* Sutter County Land Co., 81 Cal. 1, 22 Pac. 515 (1889); see also, Foundation Company *vs.* State of New York, 233 N. Y., at p. 185; and, more recently, T. J. W. Corporation *vs.* Board of Higher Education of the City of New York, 251 App. Div. 405; affirmed 276 N. Y. 644 (1937).
(53) Cunningham *vs.* City of New York, 39 Misc. 197, 79 N. Y. S. 401 (1902); affirmed 90 App. Div. 606, 85 N. Y. S. 1129.
(54) 39 Misc., at p. 201; 79 N. Y. S., at pp. 403–404.

(55) Thileman *vs.* City of New York, 82 App. Div. 136, 81 N. Y. S. 773 (1903); see, also, Des Moines Plumbing & Heating Co. *vs.* Margarian, 201 Iowa 647, 207 N. W. 750 (1926).

(56) Lentilhon *vs.* City of New York, 102 App. Div. 548, 92 N. Y. S. 897 (1905); affirmed 185 N. Y. 549, 77 N. E. 1190.

(57) The Court said: "The plan was designed to indicate the location of the walls that were to be removed and the levels and extent of excavation and the levels to which hollows were to be filled; but we think it was not intended as a basis upon which bidders were to figure the quantities of material to be removed, and that the express provisions of the contract and specifications requiring the removal of the entire reservoir structure were controlling." (102 App. Div., at p. 557; 92 N. Y. S., at pp. 901–902.)

(58) Odell *vs.* City of New York, 206 App. Div. 68, 200 N. Y. S. 705 (1923); affirmed 238 N. Y. 623, 144 N. E. 917.

(59) The Court declared: "Thus it will be seen that it was expected that water would be encountered but no one attempted to approximate the quantity. * * * Apparently the amount of water encountered was an element unforeseen by all parties, and, while one may regret the additional burden placed on the contractor, that burden arises through no misrepresentation on the part of the city, and he must bear it alone as best he may. Everything that could be done to put bidders on their guard as to the difficulties and dangers of this situation was done." (206 App. Div., at pp. 92, 93; 200 N. Y. S., at pp. 725–726.)

(60) Dunn *vs.* City of New York, 205 N. Y. 342, 98 N. E. 495 (1912).

(61) To the same effect see MacArthur Bros. Co. *vs.* United States, 258 U. S. 6, 42 Sup. Ct. 225 (1922).

(62) Cauldwell-Wingate Company *vs.* State of New York, 276 N. Y. 365 (1938).

(63) Sullivan *vs.* Sing Sing, 122 N. Y. 389, 25 N. E. 366 (1890); Molloy *vs.* Briarcliff Manor, 145 App. Div. 483, 129 N. Y. Supp. 929 (1911); Weston *vs.* State of New York, 262 N. Y. 46, 186 N. E. 197 (1933). Cf. Long *vs.* Inhabitants of Athol, 196 Mass. 497, 82 N. E. 665 (1907).

(64) Litchfield Construction Company *vs.* City of New York, 244 N. Y. 251, 155 N. E. 116 (1926); Kinser Construction Co. *vs.* State of New York, 204 N. Y. 381, 97 N. E. 871 (1912); Matter of Merriam, 84 N. Y. 596 (1881). Cf. Del Balso Construction Corporation *vs.* City of New York, 278 N. Y. 154 (1938).

(65) United States *vs.* Spearin, 248 U. S. 132, 39 Sup. Ct. 59 (1918); MacKnight Flintic Stone Co. *vs.* Mayor, 160 N. Y. 72, 54 N. E. 661 (1899); Filbert *vs.* Philadelphia, 181 Pa. 530, 37 Atl. 545 (1897); Sundstrom *vs.* State of New York, 213 N. Y. 68, 106 N. E. 924 (1914).

(66) Montrose Contracting Co., Inc. *vs.* County of Westchester, 80 Fed. (2d) 841 (C. C. A. 1936).

(67) The principal clauses relied on read as follows: "That where material other than solid rock is encountered, containing loose sand or mud, which shifts with the water contained in it, and the Contractor chooses to use compressed air to pass through it, in order to partly relieve him of the additional cost of preventing and caring for the incoming water and material or restraining it, an allowance will be made throughout the stretch where the compressed air is actually used under the above conditions of $30.00 per foot and for not more than six hundred (600)

feet of length, where actually used, regardless of whether more be used or not."

"Maximum partial compensation for labor, plant and materials under compressed air when and if actually used as specified, but not in excess of 600 ft in length and Thirty and no/100 dollars ($30.00) per foot."

(68) The Court said that the contract provisions: "* * * cannot be given the effect of completely negating the representations in the plans that the tunnel was primarily a free air tunnel. They might have been effective to limit appellee's liability if the work had required somewhat more than 600 ft. of compressed air, but in point of fact, there was 6,142 ft. to be so built. These clauses cover a variation, not a transformation." (80 Fed. (2d), at p. 843.)

(69) Golden & Son, Inc. *vs.* Marblehead, 68 Fed. (2d) 875 (C. C. A. 1934).

(70) Allen N. Spooner & Son, Inc. *vs.* City of New York, 97 N. Y. L. J. 1439 (1937).

(71) Matter of Keep, 138 Misc. 194, 245 N. Y. Supp. 321 (1930); reversed on other grounds, 237 App. Div. 377, 260 N. Y. Supp. 237; affirmed in 262 N. Y. 596, 188 N. E. 80.

(72) Meads & Co. *vs.* City of New York, 191 App. Div. 365, 181 N. Y. Supp. 704 (1920).

(73) Cauldwell-Wingate Company *vs.* State of New York, 277 N. Y. 365, at p. 377.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

BEACH EROSION STUDIES

By Earl I. Brown,[1] M. Am. Soc. C. E.

Synopsis

Items of information considered necessary in a comprehensive beach erosion study, the reasons for desiring each particular item, and some general observations on design of protective works are offered, in some detail, in this paper. The presentation is made for two reasons: (1) That engineers interested in beach protection may have the advantage of experience gained by the Beach Erosion Board, United States War Department (under the Chief of Engineers, U. S. Army), as to factors involved in a study of beach erosion; and (2) that the items of information now believed best suited to the purpose by the Board may be the subject of full and free discussion by all engineers interested.

Beach Protection Studies

The science of beach protection is comparatively new in the United States. The rapid growth of seashore resorts and the immense investment therein since automobiles and good roads have made them easily accessible have resulted in a demand for improvement and stabilization of old beaches, and development of new ones. In the past, as each seaside resort developed, local judgment determined the proper method of improving and stabilizing its beach. There resulted a wide difference in character and design of protective structures. Some have proved effective, others have been useless, and still others have done more harm than good.

Each locality presents a different problem. In all cases there is need for detailed and accurate information on local conditions. It is inadvisable to prescribe a given plan of protection for a beach simply because it has proved effective elsewhere, unless conditions can be shown to be similar. If engineers along the coasts can be encouraged to develop a general understanding of the character of information necessary for designing protective systems, and the general principles of design, a large field opens for them in undertaking beach erosion studies and designing protective works for private property owners. A suggested outline for a report on beach erosion studies is given in the Appendix.

Note.—This paper was presented at the meeting of the Waterways Division at Jacksonville, Fla., on April 21, 1938. Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **March 15, 1939.**

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The Fifteenth International Congress of Navigation which met in Venice, Italy, in 1931, discussed the question of protection of coasts against the sea, and adopted a comprehensive report drawn up by its General Reporter for

Fig. 1.—Typical Shingle Beach Along the New Hampshire Shore, North of Boar's Head

Fig. 2.—Pocket Beach Formed Against a Jetty, at Hampton Beach South of Boar's Head, N. H.

this subject, expressing its matured general conclusions. This report contains the following conclusions as to preliminary studies:

"Any plan for protecting coasts against the sea must be preceded by a careful study of the locality and all the factors acting in the formation of the

coast, such as the nature and shape of the coast, and as to how much it is exposed; the action set up by waves, by currents of various kinds, by atmospherical water and by ice; the origin and nature of the materials constituting beaches; the flow of water courses which empty themselves onto the beaches and the regimen of the mouths of such water courses; the situation and regimen of the aquiferous sheets of fresh water flowing towards the sea; the influence of new constructions on the sea, when they project from the sea."

Characteristics of Beaches

The type of information required in a study is based on these general principles, and will depend for detail on the location of the area. Conditions vary somewhat for each location. The coastal area of the United States may be divided roughly into subdivisions in each of which general conditions are quite similar: North Atlantic, Central Atlantic, South Atlantic, Florida, Gulf of Mexico, South Pacific, North Pacific, and Great Lakes.

Fig. 3.—Formation of Moriches Inlet, Long Island, N. Y., March 10, 1931

For purposes of beach erosion studies, the North Atlantic area may be considered as ending at Montauk Point, Long Island, N. Y., but including the north shore of Long Island Sound. The coast in this area consists of rocky or boulder-strewn headlands, and pocket or crescent beaches of sand or pebbles. Illustrative of these two types, Fig. 1 shows a steep shingle slope of the shore, with a narrow strip of sand at the water's edge; and Fig. 2 shows a pocket beach formed against a jetty. There are a few exceptions to this rule, such as at Salisbury Beach, and beaches along Cape Cod, in Massachusetts.

The south shore of Long Island, formed by the outwash of a terminal glacial moraine, with lagoon areas in the rear, is quite similar to the New Jersey shore and may be classed in the Central Atlantic area. This Central Atlantic type of beach (a comparatively narrow sandy barrier, broken at intervals by inlets communicating with lagoon areas in the rear) extends as far south as Cape Romain, in South Carolina. Fig. 3 is an aerial photo-

FIG. 4.—TYPICAL LOUISIANA BEACH; GRAND ISLE, LA.

graph of Moriches Inlet, Long Island, N. Y., as it was in the process of cutting through a sand barrier. South of this point, to the Florida line, the coast is intersected by innumerable tidal streams resulting in a chain of islands with sandy beaches on the ocean side and occasional marsh areas behind them. Still farther south lie the Florida beaches which superficially resemble those of New Jersey, but which, from the standpoint of beach erosion studies, are distinctly different because of a difference in climate and exposure to wave action, and the presence of limestone rock.

The beaches along the Gulf of Mexico vary considerably among themselves. They are subjected to the same type of wind, weather, and wave conditions, and, therefore, are given a separate classification. From the southern tip of Florida northward there is the inaccessible and greatly intersected Everglade area, with a total absence of beaches. Northward from Naples, Fla., to Tarpon Springs, Fla., there are barrier islands with excellent sandy beaches. North of Tarpon Springs the shore is so well sheltered from wave action because of its location and the presence of shallow limestone ledges that mangrove swamps grow to the water's edge. This condition extends to Apalachee Bay, Florida. West of Apalachee Bay to the vicinity of the Mississippi River and from Sabine Pass, Tex., westward and southward to the Mexican border there are sandy barriers with good beaches, although the island barriers of the section east of the Mississippi Delta are so far offshore that each of the two parallel beach lines presents a set of problems of its own. This is particularly true of the mainland and island beaches of the Mississippi Sound area. In

this vicinity the coast is broken by the low marshy islands of the delta, with some sandy beaches on the Gulf face. Fig. 4 shows a typical Louisiana beach on the Gulf side of a barrier island. The sand is very fine and both the foreshore and offshore are very flat. There are few dunes behind the water-line and the barrier is topped by hurricane tides.

For the purpose of beach erosion studies, the beaches in the South Pacific Ocean may be considered as those lying in the reach from Point Conception, Calif., southward to the Mexican border. The coast is comparatively young geologically, and the shores are narrow with comparatively high hills close to shore and deep water close offshore. This results in headlands between crescent beaches (see Fig. 5). The distance between headlands is comparatively great. Annual rainfall is light. During the rainy season, however, there are periods of heavy rain, the run-off from which carries sediment to the coast. This sediment forms much of the beach material.

North of Point Conception, in the North Pacific, the headlands become more frequent and the intervening beaches shorter. There is comparatively little sea-bathing in this area because of the coldness of the water, but the beaches are used as recreational areas during the summer months.

Fig. 5.—Cabrillo Beach, San Pedro, Calif., Facing North from the Breakwater

The Great Lakes area offers a different set of conditions. The Lakes are tideless, but have seiches and seasonal variations in level. They are frozen near the shore during the winter and are subject to violent storms. The bottom materials, the fetch, the slope of the bottom, currents, and ice action, differ from those on the open ocean. The Lake beaches are generally composed of a layer of very fine sand over clay or mud. The offshore is flat, and some distance out from shore the bottom changes from sand to fine clay or mud. In some sections, such as the southern end of Lake Michigan around Chicago, Ill., there are extensive sand dune areas, but in most parts the Lake shores end against bluffs of clay of varying heights.

Scope of a Beach Erosion Study

Within each area local conditions will dictate to a large degree the extent of a study and the data required. Studies on a long unbroken beach will require different information from those in the vicinity of an inlet; and a study involving a highly developed resort, such as Atlantic City, N. J., would justify a much more searching investigation than a study of a similar area which is entirely undeveloped. In determining the data to be required, the location and the economic justification for the degree of detail are first considered.

The usual requirements for a beach erosion study are an investigation of the past history of the area from all available records, and an investigation of the present conditions by means of local surveys and observations. After this information is assembled a comparison with other similar areas where there are protective works is possible, and a proper design for the protection of the area under investigation can be made. With these requirements in mind, a brief description is given of information generally secured in a study, with the reasons for, and the method of, securing it.

There are nine principal items to be investigated in considering the past history of a beach: (1) Geology; (2) shore-line changes; (3) offshore changes; (4) wind records; (5) storm effects; (6) type of protective works installed and their effectiveness; (7) volume of sand moved under different conditions; (8) tides; and (9) predominant direction of littoral drift.

Geology.—In investigating the past history of a beach the geology of the area should be reviewed primarily to determine the causes for the type of coast encountered, the source and character of the local beach material, and the geological structure in the vicinity. Even in the less extensive investigations, a brief study of the geological history is of great benefit as a guide to the more recent history. It may not be necessary to employ a geologist for the field work, but it is desirable to consult geological maps of, and reports on, the area under consideration.

Shore-Line Changes.—A comparison must be made of the shore-line changes that have occurred since the earliest authentic maps. This can be done by studying all available field sheets of the United States Coast and Geodetic Survey, local surveys, and maps of surveys of the United States Engineer Department. These data are reduced to a common scale and the shore lines, both mean high water and mean low water, for all dates are plotted so that they may be superposed on one sheet. If sufficient surveys are available, covering a long period of time, a "good picture" is afforded of the progressive changes that have occurred. Along many parts of the New England and New Jersey coast excellent surveys have been made by the U. S. Coast and Geodetic Survey, extending back to 1839. Unfortunately, this is not true for all the seacoast. At some places the available surveys are irregularly or remotely spaced in point of time. For example, in a study at Daytona Beach, Fla., the earliest accurate survey that could be relied upon for shore-line changes was that of 1872–1874; the next one was made in 1928, a lapse of 54 yr. An office comparison of shore lines as outlined is made for all studies. Where a study warrants more detail, a survey showing the present location of mean high and

low-water shore lines is made, and the results are superposed on a map of the old shore lines to show the changes to date.

Offshore Changes.—A history of offshore changes is secured in much the same way as that of shore-line changes. Copies of hydrographic field sheets of the offshore area, obtained from the U. S. Coast and Geodetic Survey and the U. S. Engineer Department, are reduced to a common scale and curves for 6-ft, 12-ft, and 18-ft depths superposed on one sheet for comparison. Where the changes between surveys are considerable, separate work sheets on a relatively large scale for each depth-curve comparison are prepared and then consolidated on one sheet for the report. In case of an extensive study, the 24-ft, 30-ft, 36-ft, 42-ft, and 48-ft curves are sometimes added. Generally, an offshore survey is required for the study in order to determine the present position of the curves to be compared.

The purpose of comparison of offshore depth curves is to determine if progressive changes are occurring. Past changes will have a bearing on probable changes to be expected along the shore in the future and will thus affect the design of proposed protective works. If the changes show a progressive and rapid movement shoreward of the depth curves, indicating a deepening of the offshore area, it is evident that severe wave action close to shore may be expected, which may cause increasingly rapid erosion of the beach. Conversely, a progressive shoaling of the offshore area will indicate a probable lessening of destructive wave action.

In studies of larger scope, it is desirable at times to study general trends on the bottom for distances of 5 to 10 miles offshore. This study is made from hydrographic field sheets of the U. S. Coast and Geodetic Survey. Ranges are selected along the area to be investigated on which soundings were made on several of the surveys. Composite profiles along these ranges are plotted for each survey and compared on each range. On the Atlantic Coast it has been possible, in several instances, to follow the seaward or shoreward trend of the depth curves to depths of 70 ft. On the California coast where depths increase rapidly offshore, there were bottom changes, in 300-ft depths, of as much as 18 ft between the 1869 and 1930 surveys. Such a large difference can scarcely be attributed to errors in soundings or movement of littoral drift, but may well have been caused by slipping or faulting during an earthquake. Investigation of this kind is most effective and profitable of results when done on a large scale.

Wind Records.—The next step in investigating the past history of an area is a collection of wind and weather records. There are three available sources for past weather records: The United States Weather Bureau, the Coast Guard Stations in the area under investigation, and airports. Generally, the nearest first-order station of the Weather Bureau can furnish quite accurate records since 1923. Prior to that time the instruments used were not standard and there is some doubt as to the accuracy of the records. Where the nearest Weather Bureau Station is some distance from the area under investigation it is well to take the records from the nearest Coast Guard Station. These generally consist of readings every four hours on the wind direction and velocity. The readings are approximate and must be reduced to a usable form for

a study. This involves considerable office work, but affords a clear idea of general wind conditions in the area, particularly the duration and intensity of storms and of prevailing winds.

Some difficulty may be encountered in interpreting and using wind diagrams furnished by outside sources. Two types of wind-roses have been developed for beach erosion studies (see Fig. 6), which make it possible to determine the

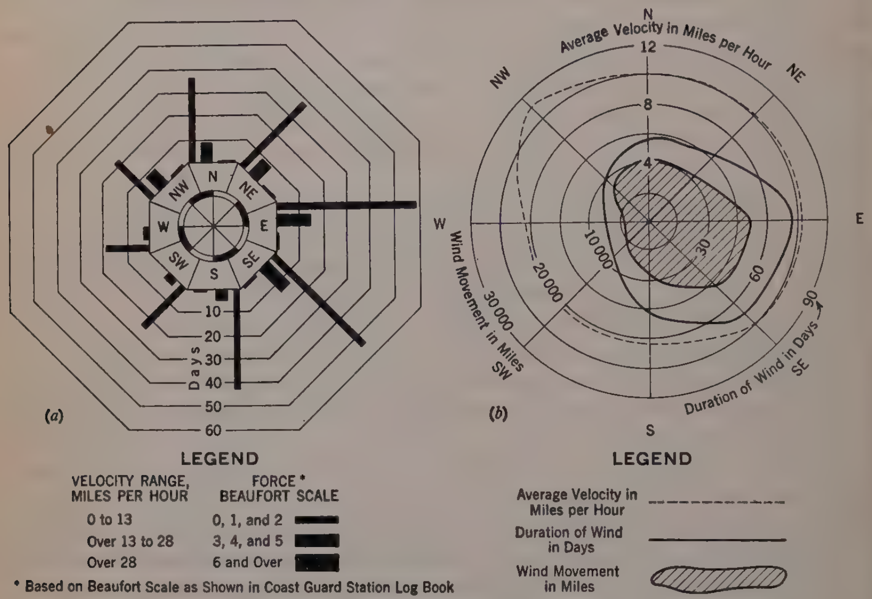

Fig. 6.—Typical Wind Diagrams

prevailing wind movement, the average velocities from each direction, and the direction of predominant storm winds. If, from an examination of the data, it is apparent that there is a great seasonal variation in wind direction and intensity, and if the detail of the study warrants, such wind-roses should be plotted for each month in the year over the entire period covered, and a second series should be made showing the average for each month, and, finally, the average for the entire period. Another source of wind information is the Hydrographic Office, U. S. Navy Department. Its records show prevailing winds and their intensities in 5° areas offshore as reported by ships.

The study of past wind and weather conditions is important when correlated with the shore-line and offshore changes and reports of the effects of storms on the beach. By affecting the direction of wave approach, winds play an important part in determining the direction of predominant littoral sand movement. Wind-blown sand carried ashore plays an important rôle in dune building and lateral sand movement along the shore. By examining the history of winds and storms over a considerable number of years, it is possible to obtain some idea as to why certain changes have occurred and what to expect in the future, particularly with respect to sand movement, the direc-

tion, duration, and intensity of storms, and the time of year when storms may be expected. The extent to which the investigation of wind and weather is carried will be determined by the general detail warranted for the study. It should not be ignored in any study.

Storm Effects.—In addition to the study of winds, a history of the general effect of intense storms in the area has been found of value. The information may be obtained from long-time residents, from newspapers and other records, and from photographs. Particularly desirable is the extent of erosion or accretion caused, the apparent effect of any protective works or other structures in the area, and the height to which the tides rose during storms. Since there is danger of exaggeration, in any eye-witness description of storm action, such description must not be relied upon too heavily. Photographs showing conditions before and after a storm are among the most reliable sources of information.

Existing Protective Works.—In the history of a beach area, a complete and accurate record of any protective works which have been installed must be secured. This should cover the location, profile, and date of construction of each, with a history of its effect upon the beach and its length of life. It is usually possible to get the plans and specifications if the structures were built by a municipality, State agency, or the Federal Government. In all cases, it is possible to note the present condition, the effect on the beach, and the probable life. Where long structures such as jetties have been in place a considerable period of time, changes in the offshore areas as well as those at the shore line should be considered.

In a detailed study, the effect of all structures projecting into the sea on the shore line over a number of years should be followed. Changes in the location of the shore line and the depth curves are valuable indications. Frequent or yearly hydrographic surveys should be available with sounding ranges fairly close together and profiles extended back from these ranges over the area of dune sand. From such surveys it is possible, by using a similar grid for each year, to compute the volumetric sand changes in the shore and offshore area to the limit of the surveys. This is being done at several places along the Atlantic Coast with very interesting results.

In evaluating the effect of groins or jetties it is necessary to discriminate between the effects of a single structure and that of a system. The supply of sand that may be trapped along a given shore may have been sufficient to show satisfactory results with a single structure, but may not suffice for prompt results for a system of structures. An artificial supply of sand may be required in such a case.

Tidal Data.—It is advisable to learn about the tides in the area under investigation. The time and height of tides, the mean range, and spring and neap ranges may be obtained from tide tables published by the U. S. Coast and Geodetic Survey. In addition to these mean values, it is important to secure information on the extremes under storm conditions, since any design of protective works must consider these extremes. This information can be obtained from the U. S. Coast and Geodetic Survey for points where tide gages

have been established. In other cases it is necessary to refer to the U. S. Coast Guard or other local records.

There are no tides on the Great Lakes but there are minor irregular variations in level known as seiches and fairly regular seasonal variations between winter and summer, averaging between 1 ft and 2 ft. During the winter the Lake levels are low and during spring and summer they are high. In addition to the seasonal variations there are long-time cycles of higher waters and lower waters. In the case of Lake Huron, for example, the seasonal variation since 1860 averages 1.2 ft, the cycle of higher waters averages 14 yr and the maximum difference between higher high waters and lower low waters has been 6.30 ft. It is apparent that variations of this nature must be considered in designing any protective works on the Lakes. The United States Lake Survey has kept accurate records of seasonal variations in all the Lakes since 1860.

Fig. 7.—A Typical Florida Beach; Lake Worth Inlet, Fla.

Littoral Drift.—The last item to be considered under the past history of an area is the direction of predominant littoral drift. This information is most important in the design of a protective system. From the shore-line changes, particularly near an inlet or jetty, and from changes in offshore depths, the direction of preponderant movement of the littorally drifting sand may be determined. The direction of growth of a point, the migration of an unimproved inlet, or the accretion on one side of a jetty, will give good evidence of the direction of predominant sand movement. In Fig. 7, for example, littoral drifting sand is piled against one side of a jetty entrance to the detriment of the beach on the leeward side. Inferences from an accumulation of

sand against either side of a short groin at any particular time of year may be erroneous since, at many places, there are seasonal reversals of the direction of sand movement. The direction of wind and of wave approach plays an important rôle in directing sand movement along the shore and in seasonal reversals. If the detail of the study permits, observations as to the direction of littoral drift should be continued over a period of a year to determine these seasonal changes.

Comment.—In a beach erosion study, the foregoing nine items dealing with the history of the beach are all considered. This is relatively expensive, since considerable office work is involved. Caution is given against singling out one of the items of investigation and basing conclusions and recommendations on it. For example, the history of offshore changes may show a considerable movement seaward of the curves for 6-ft, 12-ft, and 18-ft depths indicating shoaling of the offshore area. Normally this would afford greater protection to the beach. However, there might be a near-by longshore channel with comparatively high current velocities which could cause a serious erosion of the beach. In all studies it is important to complete all the items of investigation, to correlate them, and to give due consideration to their accuracy before arriving at conclusions and prescribing a remedy.

Field Work

After having decided upon the items involving historical phases of the study, a plan of field work should be drawn up to ascertain present conditions. The location of the beach, the problem to be solved, and the availability of funds will dictate what field work should be included.

Eleven major items of field work must generally be considered, many of them being closely related and all of them involving varying degrees of detail, namely: (1) Character of the beach; (2) character of the offshore area; (3) currents; (4) tidal prism of inlets; (5) tides during the field work; (6) littoral movement of sand; (7) present condition of protective and other works, and their apparent effectiveness; (8) wave direction, velocity, and height during current and sand movement observations; (9) wind conditions during the course of field observations; (10) sand analyses; and (11) periodic aerial photographs.

Character of Beach.—The usual procedure for small studies, in determining the characteristics of the beach, is to measure profiles extending across the dune line to wading depths beyond mean low water. These profiles are spaced from 200 to 500 ft apart along the entire front under investigation and are perpendicular to the shore. When plotted they furnish information on the height of dunes, the general slope and shape of the area above high water, the location of mean high-water and mean low-water lines, and the slope of the foreshore. These are all factors in the design of protective works and should be included in all studies. Sand samples are taken at the mid-tide elevation on the beach at each profile. These samples are analyzed for median diameter, specific gravity, porosity, and shell content. Since the degree of wetness of beach sand greatly affects its physical properties, a determination of its moisture content, and of the source of the water, whether the sea or ground-water, is coming to be considered a matter of some importance.

There is a possibility of a relationship between the slope of a beach and the characteristics of the sand composing it. In general, it appears that the greater the median diameter the steeper will be the beach slope; and the smaller and more uniformly graded the sand, the flatter the slope will be. Although it is not of great importance to any particular study this information may be included for statistical purposes.

The analyses of the sand are of value in determining the source of supply of the beach sand. Where this is of material interest to the study, samples are secured from all suspected sources for comparison with the beach samples. Complete petrographic studies are usually made only in special cases.

In the case of extensive and detailed studies, topographic maps of the area should be made to show dune location and height, and the character of the country some distance landward of the beach. Aerial photographs are useful in connection with these maps to show the extent of developed areas and buildings.

Character of Offshore Area.—The under-water topography can be obtained from a general hydrographic survey of the area or by extending the shore profiles to depths of 18 ft to 30 ft below mean low water. For smaller studies it is usually not necessary to extend these lines beyond 1 500 ft from shore, or to more than a depth of 18 ft below mean low water. These profiles or hydrographic surveys are important in any study to show the offshore slopes, and the presence of offshore bars and reefs, and to afford a comparison of present conditions with those shown in previous surveys. The existence of an offshore bar or reef, if close to shore, will affect the design of groins.

In more detailed studies it is necessary not only to know the under-water topography, but also the character of the bottom materials, since the bottom may be the immediate source of material reaching the beach. Bottom samples should be dredged on the intersections of a grid system and then mechanically analyzed and leached with hydrochloric acid. The results will show the character of bottom material and shell content of the samples. Borings may be used, if desired.

In cases involving inlets the system of profiles is usually not sufficient because, in addition to the profiles, it is desirable to have a sufficiently detailed hydrographic survey of the inlet to develop the entire outer bar, the inlet channel, and the inner bar.

Currents.—A study of currents in the ocean comparatively close to shore is of importance in determining the direction of sand movement along the beach and bottom. In measuring currents offshore, it has been found convenient to use floats covering a complete tidal cycle, to determine any changes caused by difference in the phase of tide. Except immediately adjacent to the shore, and at inlets, such currents are usually moderate or slow.

At the same time that the offshore currents are being measured by means of sub-surface floats, the currents on the same lines close to shore should be measured. From the limit of wave up-rush to wading depths this can be done conveniently by timing the longshore movement of the water by surface floats or by the use of colored liquid. The latter is preferable since it registers the actual movement of the water through the wave up-rush and back-wash and

is not affected by wind. Potassium permanganate or washing blueing have been satisfactory for this work. The measurements of current near the shore must continue over the entire time during which the offshore currents are being measured—one full tidal cycle.

For more detailed studies warranting a close determination of the bottom currents a specially designed current meter[2] has been developed which can be operated at any reasonable depth from a boat offshore. These meters are very satisfactory and are much to be preferred to sub-surface floats since they register the actual bottom currents—those that move the sand. Measurements must be made at a reasonable number of points along each range line to be covered, and, at each point, must cover a complete tidal cycle.

In offshore current work, with the present equipment, it is impracticable to make measurements except during fairly calm conditions. It would be a decided advantage to know the relation between calm-weather currents and sand movement offshore and those prevailing during storms, and also the depth in which sand movement occurs during storms.

Tidal Prism of Inlets.—If the study involves the fixation or improvement of a tidal inlet, additional current measurements of a somewhat different type are required. A careful cross-section of the gorge is made and currents are measured at sufficient points along its line to enable making an accurate computation of the tidal prism. This requires that the measurements at each point extend over a complete tidal cycle. A tide-gage should be established at the ocean entrance to the inlet and another at the lagoon end. Where staff-gages are used they should be read every 15 min during the period of current measurements. If the study warrants, staff-gages should be established and read at critical points in the rear areas of the lagoon to determine the propagation of the tidal wave in the lagoon. Since the currents in the inlet are tidal and persist longer than oscillations due to wave action, any good type of current meter will be found satisfactory. The data are used, and conclusions drawn therefrom, as described by the writer in a paper published in 1928.[3]

Tidal Data.—Many of the items of field work are closely related to others. One of these items is the securing of accurate tidal data. The times of high and low waters and the average ranges of mean tides and spring tides, as given in tide tables, cannot be used for the reduction of soundings made along profile lines in the ocean, nor for correlation with the current measurements. They are means of long-time observations. Wind and weather have such an effect that the actual tide rarely, if ever, follows exactly the mean, as given in the tables. Therefore, whenever a study involves ocean profiles, ocean currents, a study of sand in suspension, or of inlets, it is important to establish a tide-gage to register ocean tides at the area under investigation. These observations must be made continuously during the time of field work.

Littoral Drift.—Direction of sand movement is determined, if possible, from historical evidence. It should also be studied in the field. If the importance

[2] "Effect of Turbulence on the Registration of Current Meters," by the late David L. Yarnell and the late Floyd A. Nagler, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 95 (1931), pp. 800–806.

[3] "Inlets on Sandy Coasts," by Earl I. Brown, *Proceedings*, Am. Soc. C. E., February, 1928, pp. 505–553.

of the study warrants it, a field determination of the volume of sand movement in the water under calm-weather conditions at the time of the study may be made with a fair degree of accuracy. This is done at the same time and on the same lines as the current measurements. Samples of water with the sand

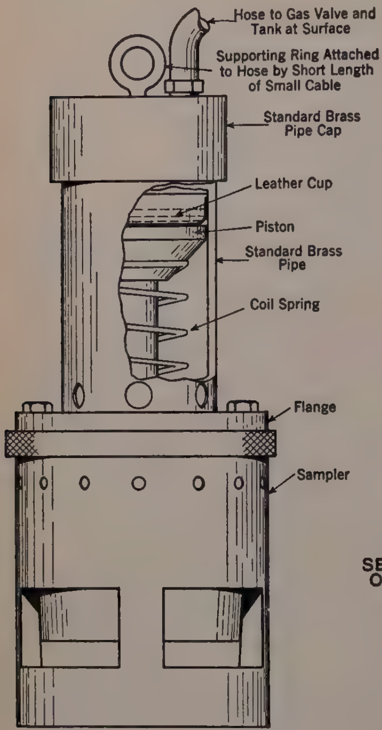

FIG. 8.—DEEP-WATER SAMPLER

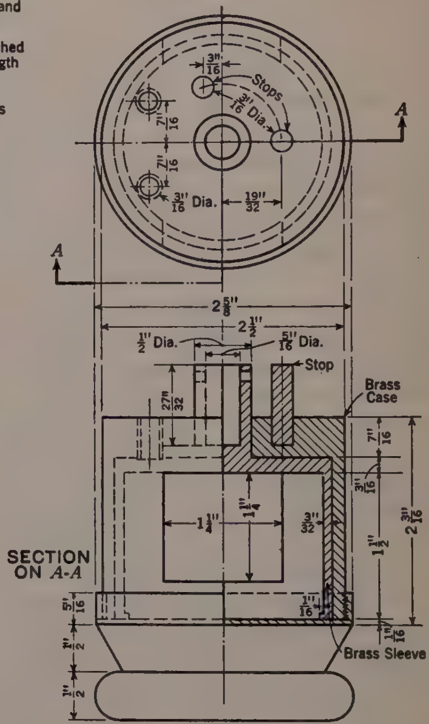

FIG. 9.—HAND SAMPLER

in suspension are taken from the bottom upward to a point where there is none. Fig. 8 shows a deep-water sampler in a partly open position. If currents are being measured by means of sub-surface floats, the boat releasing the floats takes several samples at each point and additional samples each time the floats are set on the range. Samples are taken in 1-ft increments above the bottom point until no sand is noted in the sample. This process must be continued throughout the full tidal cycle as in the case with the current measurements. If bottom currents are being measured with the current meters, samples are taken periodically throughout the time of meter readings at each station. The samples of sand in suspension should be secured for each line upon which currents are being measured.

In addition to the aforementioned offshore samples, samples to show percentage of sand in suspension must be secured from the limit of wave up-rush to the limit of wading depths on the same lines and covering the same period as the offshore samples. A hand sampler, similar in general principle to the offshore sampler, is used for this work (see Fig. 9).

The sand is filtered from all samples on the site and sent to a laboratory where the percentage of sand in suspension is determined. From the results thus obtained, curves showing the average percentage of sand in suspension at each point and the upper limit of suspended sand may be plotted. These data, combined with the results of the current measurements, permit rough computations of the volume of sand in suspension moving across a given line perpendicular to the beach from the limit of up-rush to the profiles. This method necessarily neglects the sand that is moving along the bottom by saltation, but there will be no such movement unless the current velocity is greater than the critical. As in the case of the current measurements it is impracticable to secure these samples except under fairly calm conditions, and, therefore, no comparison has been made between sand movement during normal and storm conditions. The investigation of the Beach Erosion Board of this sand movement thus far indicates that during calm weather approximately 80% of the sand movement is in depths of water less than 6 ft.

On many beaches the movement of sand by the wind may be of importance. Offshore winds have little dry sand-moving capacity, but winds on shore or parallel with it may move it in large volumes. Observation has shown that dry sand is not moved by wind having a velocity less than 11 to 12 miles per hr, but above that rate movement increases rapidly with wind velocity. The rate of wind-blown sand movement, when desired, may be measured in a suitably designed trap.

Observation of Protective Works.—In order to determine the effectiveness of protective works a close examination should be made to compare their present condition with the plans and specifications. It is well to check whether they were built in accordance with the specifications or modified. Careful notes should be made as to the spacing, length, and height of groins, and the apparent effect on the beach of these factors. Photographs of the beach for some distance on either side of the protective works are of value in formulating conclusions and in planning new protective works. Especial care should be taken to observe the effects of existing protective works which project from the coast line.

Wave Data.—Whenever observations on currents in the ocean or sand in suspension are made, careful readings of wave data should be made and recorded. The following data should be observed and recorded both for the shore and offshore stations: Wave height, period, length, velocity, and direction of approach. Accurate determination of these five factors is difficult, but a close approximation may be made by observing and averaging ten waves at intervals of 15 to 30 min. Offshore the wave height may be measured by observing the rise and fall on a lead line held with the lead just resting on the bottom. The period may be clocked, of course, and the velocity timed from bow to stern of an anchored boat. The length may be computed from the period and velocity, and the direction of approach read by compass. Near the shore the measurements are easily obtained if there is a pier close by. If no pier is available, piles may be driven at known intervals perpendicular to shore. This is comparatively expensive and is not warranted in a small

study. Wave height, direction of approach, and period, which are the most important factors, can always be read from shore without the pier or piles.

Waves are a most active agency in moving sand along the beach. The data obtained from the aforementioned observations should be scrutinized carefully in conjunction with the currents, the sand in suspension, and the wind and weather conditions for some time prior to the observations. An examination of the wind-rose and the history of direction of predominant sand movement correlated with the wave data, currents, and sand in suspension, will assist in forming an opinion of the erosive forces and in designing a system of protective works.

Wind Data.—As accurate a record as possible should be kept of the wind direction and intensity during the course of the field work. The cost of this item is small, and it is useful in determining the effect of wind on the direction and characteristics of the waves.

Sand Analyses.—In the design of a bulkhead or groin, or for the dredging of a channel through a spit or outside bar, it is important to know the character of the sub-surface materials. In the normal beach-erosion study it is only necessary to know whether there is any rock or hard material that will make the driving of piles difficult or impossible. This information may be obtained by probing or by jetting if the piles are to extend to a considerable depth. To secure a more accurate idea of the character of the material, test pits above the high-water line may be dug at intervals along the beach and a log made of the materials.

To determine the character of the offshore sub-surface, wash borings may be made below the bottom from the shore out to depths of 40 ft, or more, below mean low water.

Aerial Photographs.—Fig. 10, a view of Moriches Inlet, Long Island, may be compared with Fig. 3, taken one month earlier. In Fig. 3, the rapid on-rush of water from the bays in the rear shows the inlet in process of forming. Fig. 10 shows the effect of tide action in forming a beautiful delta inside the inlet. Such aerial photographs are excellent for a continuing study, such as when it is desired to follow and record progressive changes like the migration of an unimproved inlet on a sandy coast, or the effect of accretion or erosion near jetties. For any study, they have an advantage in showing the general character of the shore. Unless they are already available the cost of securing them is prohibitive for a small study. Through the co-operation of the United States Army Air Corps, the Coast and Geodetic Survey, and the Geological Survey, the Beach Erosion Board listed the areas of the coast that have been photographed.

Comment.—In outlining the field work for a beach erosion study, all the items mentioned should be considered and evaluated with regard to the particular problem to be solved and to the extent of detail justified by the funds available. Some items may be eliminated and others may be greatly curtailed. In this case as in many other engineering problems, the experience of the investigator and his judgment play an important part since it is impossible to prescribe fixed rules. One general rule may be given: An engineer laying out a beach erosion study must not allow his desire for quick results to

curtail the work necessary to secure sufficient data upon which to base a plan of protection.

After the data decided upon have been secured, each item should be developed separately and then correlated with the others to obtain a true picture of the forces at work and their effect. The wind-roses should be examined

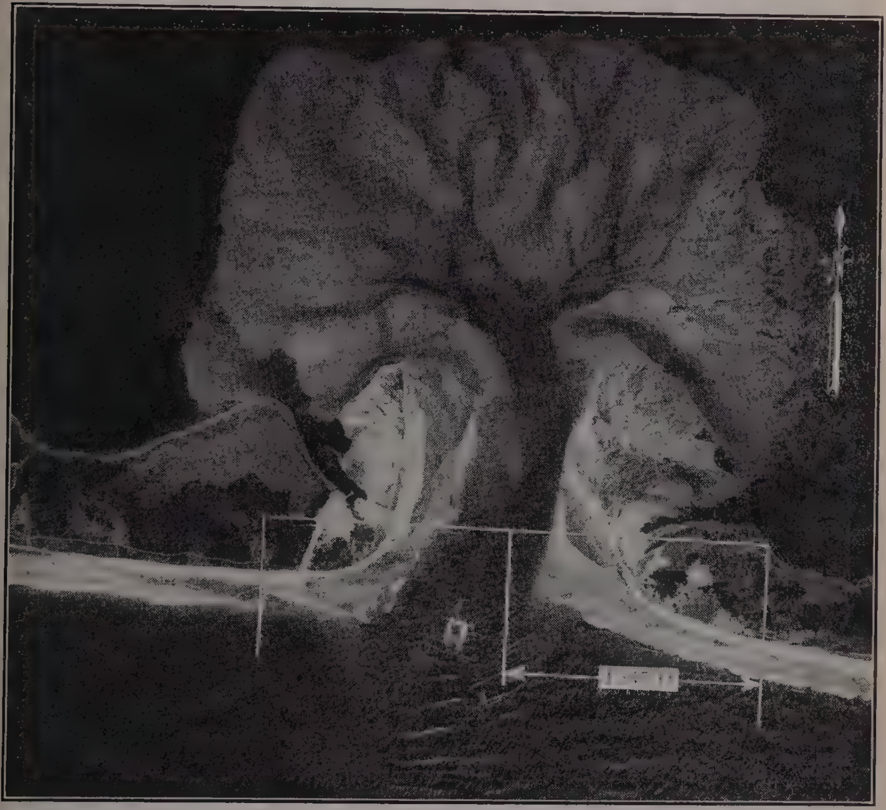

FIG. 10.—MORICHES INLET, LONG ISLAND, N. Y., SHOWING DELTA FORMED BY ONE MONTH OF TIDE ACTION

carefully and compared with the wave data to determine the possible effect of the winds upon the height and direction of approach of the waves. Study the current measurements, the sand in suspension, and the tides, to note the respective effects on heights and time. The long-time wind-rose should be studied in connection with the history of beach and offshore changes and the effect of storm action. From past beach changes as effected by past weather conditions and past storms an idea may be formed as to the future changes to be expected.

OBSERVATIONS ON DESIGN OF PROTECTIVE WORKS

Bulkheads.—The problems to be considered in preparing a design for a simple bulkhead and groin system may be summarized briefly. Assume the

case of an area to be protected on a long straight stretch of beach distant from the influence of an inlet or change in direction of the shore. Assume that the solution may call for both bulkhead and groins. First, consider the placing of a bulkhead. The factors governing this problem are the location and character of the developed area in the rear, the presence or absence of dunes, the general slope of the beach, the width of recreation beach desired in front of the bulkhead, the character of wave action, and the character of the subsurface material. This information can be obtained from the topographic map or the profiles, the wave observations, history of storms, and the probings or borings.

The line of the bulkhead should be kept as straight as possible. Where slight changes in direction of the beach require a change in direction of the bulkhead, this should be accomplished by a gradual smooth curve, not by sharp angles. Sharp angles or set-backs in a bulkhead system generally cause a concentration of the wave forces with resulting erosion.

Determination of the proper height for a bulkhead requires the consideration of four factors: (1) The character and extent of the development in the area to be protected; (2) the elevation and character of the ground in the rear; (3) the maximum heights and frequency of storm tides; and (4) the probable effect of the groin system in accumulating sand.

Where the development is extensive, warranting the expenditure of considerable sums to afford complete protection against over-topping by the highest storm tides and waves, a massive sea-wall is suitable. An example of this type of protection is the Galveston (Tex.) sea-wall. At other locations, with extensive developments, due to the character of the storm tides and wave action, the volume of sand on the foreshore, and the height of land in the rear, a bulkhead would answer the purpose at a considerably lower cost. Economic considerations determine whether the height and strength of the protective work shall be such as to afford complete protection or whether protection should be afforded against ordinary storm tides only. The lesser protection against ordinary storm tides and waves may result in some destruction during exceptional storms but will prevent complete disaster.

Assume that the development warranted protection only against ordinary storm tides, and that the land in the rear was comparatively low. In determining the proper bulkhead height the probable effect of the groin system in accumulating sand to act as a buffer to storm waves should be considered. If a large quantity of sand is moving along the shore, to be trapped by the groins, this accumulation will act as a buffer to trip the storm waves and prevent their breaking over the bulkhead. The average frequency, duration, and direction of storms should be studied to determine whether one storm might be expected to remove all the sand accumulated during normal conditions, and whether recurrence of storm conditions might be expected before normal conditions have restored a sand supply between the groins. Where conditions are bad, the sand supply cannot be counted upon to act as a buffer to the storm waves, and the top of the bulkhead should be placed at an elevation equal to the average height of the highest yearly storm tides plus wave heights, omitting from consideration extremely infrequent hurricane tides. If the sand

accumulation can be counted on to trip the waves before they reach the bulkhead, the bulkhead height may be reduced by the average wave height.

The details of design of the bulkhead will depend upon the character of the sub-surface material and on the probability of the bulkhead being subjected to the direct force of the breaking wave. The length of piles should be such that they will have about two-thirds penetration when the beach is cut down to the point where the low water is at the foot of the wall. Piles should be tied back so as to insure against failure by pressure on the land side when exposed to low water on the ocean side and should be braced to resist wave action on the ocean face. It is important in the design to insure imperviousness to protect the back-fill. The leaching of the back-fill through a bulkhead is a common cause of failure. Consideration should be given to providing release for the water topping the bulkhead during storms so that property in the rear is not damaged.

Where groins may be expected to trap sand to form a beach, the bulkhead is simply a last line of defense and an insurance against serious property loss during exceptional storms. Where the land in the rear is fairly high and well consolidated, the average storm action is not severe, and there is a sufficient volume of littoral drift to insure filling the groins, the bulkhead may be omitted. The groins should then be well keyed back into the high land in the rear. In omitting the bulkhead, it must be realized that since there is no insurance against damage by flanking, immediate and costly repairs may become necessary.

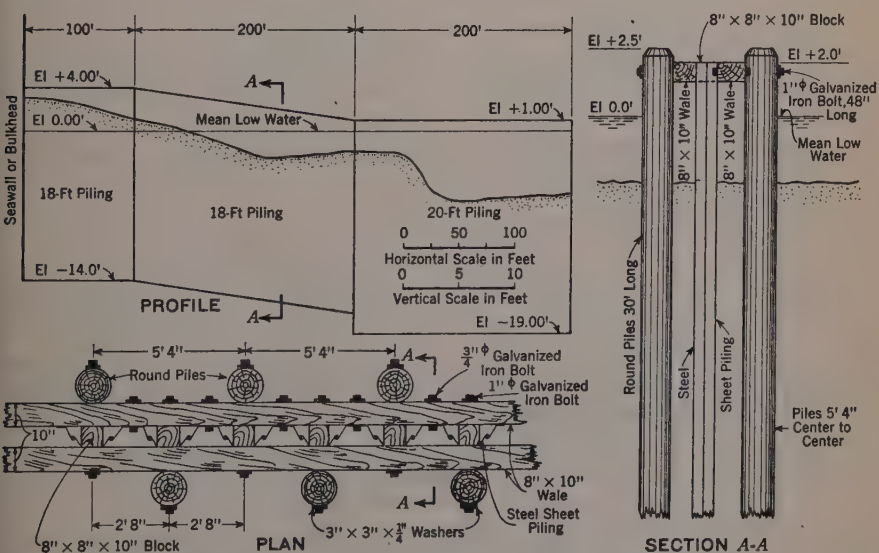

FIG. 11.—TYPICAL GROIN

Groins.—In the design of a groin the profile should be based as nearly as possible on maintaining or restoring natural conditions (see Fig. 11). An examination of profiles of beaches measured after a considerable period of

normal conditions shows, in general, a flat horizontal berm above the high-water line; then there is a sloped section extending from this upper berm to a point below low-water line; and, finally, a more gentle under-water slope. The profile should follow these variations with modifications made necessary by the difficulty of construction in the ocean. Fig. 11 shows an inner horizontal section, a connecting slope, and an outer horizontal section.

The inner horizontal section must be securely fastened to the bulkhead or must be well keyed into the high land in the rear. The top of this inner section is at the average elevation of the natural berm over the entire area to be protected. The length of this horizontal inner section is determined by the requirement of providing an adequate recreation area for the number of people expected to use the beach. Here, economics influences the "picture." If the demand is for a wider upper beach or berm than may be expected from natural accumulation, the groins may be filled artificially by sand pumped from areas in the rear or from the sea offshore. Sometimes, sand is transported from a selected source.

The angle of slope of the intermediate section is made slightly less than the average natural slope of the beach in order to encourage as flat a beach as possible. Since this section joins the inner and outer horizontal sections its length will be fixed by its slope and the difference in elevation between the two horizontal sections.

The top of the outer section is usually fixed at the elevation of mean sea level. This is a compromise with the ocean forces and the difficulties of construction. As far as the ocean forces are concerned it would be well to keep the top of the outer section of the groin below mean low water in order to check the wave currents, but not to oppose directly the force of the wave above the still-water level where its maximum force is exerted. The practical difficulty of construction below the low-water line in the ocean makes this uneconomical. Several experimental groins at Palm Beach, Fla., constructed in 1936, have alternate piles driven to mean low water with the top of the others at mean high water.

The length of the outer horizontal section of the groin will depend on the offshore conditions. If there is a reef or fairly permanent offshore bar within a reasonable distance of shore, the groin should extend across the longshore channel inside the reef or bar and end on the crest of the reef or on the outer face of the bar in depths of about 6 ft. If the offshore slope is fairly uniform the groin is usually extended to depths of 6 ft below mean low water. If the sand supply moving along shore is large enough to indicate a decided accretion between the groins and a material movement seaward of the normal low-water line this depth may be exceeded. Since about 80% of the littoral drift under normal weather conditions occurs shoreward of the 6-ft depth curve, and since observation shows a depth of about 6 ft at the ends of many successful groins, the 6-ft contour has been selected as the proper location for the end of the groins.

The detail of design of the groin will depend on the exposure to wave action and on the material used. Groins should be made sand tight with sheet-piles well bound together by wales and supported by long piles to resist

the wave action against them. The sheet-piles should have approximately two-thirds penetration under the most severe erosion expected to occur. In some locations it was unnecessary to use long supporting piles but their omission is not recommended unless good penetration in hard material affords a natural anchorage below which erosion cannot go.

In determining the spacing between groins there is a general rule, obtained from many observations of groin system: The ratio between the length of groin and the distance to the next groin should be between 1 to 1 and 1 to 3. Spacing closer than 1 to 1 does not injure the beach, but is never economical. Spacing greater than 1 to 3 appears to be ineffective in holding a good beach. Within these limits the spacing is determined by the direction of approach of the storms causing the most severe erosion on the beach. After the length of groin has been fixed by offshore conditions, draw a line through the end of the groin parallel with the direction of the storm approach. The projection of this line on the line of the bulkhead will determine the proper space to be allowed between groins. The spacing thus determined, of course, will not always fall within the limits previously set. If not, the nearest limit should be used. This method of determining spacing is not so haphazard as it may seem from a brief description. The more directly onshore the wave action is, the less will be the littoral movement of sand under the influence of the wave forces, and the less serious the erosion. In the case of storm action parallel to the coast the resulting waves do not travel parallel, but the shoreward ends are retarded so that the wave reaches the shore at an angle of about 16 degrees.

The bulkhead must be protected securely at each end against flanking by providing substantial wing-walls into solid land. The danger of increased, or of continued, erosion beyond protective works must be recognized. Numerous methods have been tried both in Europe and in the United States to prevent this cutting back beyond the ends of protective systems. In some cases a progressive reduction in length of groins toward the ends has proved successful whereas in other cases it has not been effective at all. Normally, the greatest erosion occurs at the end of the system away from the direction of sand drift. In some of its designs of protective works, where the littoral sand movement is small or where immediate results are desired, without waiting for slow natural accumulation, an artificial supply of sand can be provided to the beach by dredging from the open sea, or from near-by sheltered waters, if available, or by other means, if required. In many cases dredging offers a quick and inexpensive method of compensating for the normal losses of sand from beaches. Artificial supplies of sand must be renewed from time to time, but the method is worthy of consideration in preparing most designs.

Conclusion

Considerable research and study remain to be done before the problem of beach protection can be reduced to a definite science. There is further need for experiments in the ocean on currents, waves, and sand movement; by model experiments in its wave tank; by investigation of materials used in protective works; by periodic aerial photographs of certain areas; and by continued examination of the effect of existing structures.

APPENDIX

Outline for Reports on Beach Erosion Studies

I.—Scope of Investigation.—

- (*a*) Authority.
- (*b*) Purpose.
- (*c*) Outline of Study:
  - (1)
  - (2) etc.
- (*d*) Inspection by Board.

II.—Location and Description.—

- (*a*) Location.
- (*b*) Geology.
- (*c*) Land areas.
- (*d*) Existing projects or improvements.

III.—Field and Office Studies.—

- (*a*) Surveys.
- (*b*) Comparison of Maps:
  - (1) Shoreline Changes:
    - (*A*) Present topography.
    - (*B*) High water.
    - (*C*) Low water.
  - (2) Depth Contour Changes:
    - (*A*) Present hydrography.
    - (*B*) 6 ft.
    - (*C*) 12 ft.
    - (*D*) 18 ft.
- (*c*) Composite profiles.
- (*d*) Beach profiles.
- (*e*) Current observations.
- (*f*) Tides.
- (*g*) Sand in suspension.
- (*h*) Sand Analysis:
  - (1) Beach.
  - (2) Offshore bottom.
- (*i*) Borings:
  - (1) Beach.
  - (2) Offshore.
- (*j*) Winds:
  - (1) Diagrams.
  - (2) Analysis.
  - (3) Storm conditions.

IV.—Improvement Desired.—

V.—Discussion.—

VI.—Plans of Improvement.—

(*a*) Plans.
(*b*) Specifications.
(*c*) Estimate of cost.
(*d*) Comparison of plans.

VII.—Recommendations.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

FLOOD-PROTECTION DATA[1]

PROGRESS REPORT OF THE COMMITTEE

1.—Recapitulation.—In presenting this progress report a brief recapitulation seems in order. At the time this Committee was re-constituted in 1934 there existed a dearth of published data relating to floods not unlike that which confronted its predecessor Committee appointed in 1922. Not only were flood data of the kind needed by the practicing engineer scarce, but such as had come into print were often not dependable or in usable form, chiefly for lack of systematic research and for lack of systematic presentation. This situation has become reversed, and, within the past two years, a mass of information pertaining to floods in all parts of the United States has appeared in print, most of it in excellent form for use. Additional printed reports are in preparation covering the more recent flood events. More than half a dozen agencies, State and Federal, are now engaged in collecting and publishing such data. The outlook is that the agencies so engaged will continue these activities.

2.—This Committee, while expressing gratification at the progress so made, senses, nevertheless, that much remains to be done in the matter of providing the profession with working data. With the advent of extensive information on floods it now becomes its duty to stress the need for making the fullest possible use of this information. The more important phases of these two problems form the theme for this report.

3.—Historic Floods.—Early flood occurrences have ceased to be the subjects of doubt and ridicule. Well-directed research work has brought to light much tangible information. In addition, the great floods of the past few years have lent credence to the great floods of the past. For instance, the floods of 1762 and 1763 on the Ohio River at Pittsburgh, Pa., the accounts of which were obtained from letters preserved in the British Museum, written by commanding officers at Fort Pitt, no longer are difficult to believe. That of 1763, which by reference to the "Block House" attained a height of 41 ft on the present gage, was until March, 1936, the highest flood chronicled. The flood of that year rose to 46 ft, and although there is no doubt that undue encroachment on channel and flood plain by the works of Man had its contributory effect in building up this stage, the 1763 flood stage now stands unchallenged. It seems not out of place here to point out that even the "Deluge" described in the Book of Genesis some years ago ceased to be legendary, thanks to painstaking research by the University of Pennsylvania, in collaboration with the

[1] Presented at the Annual Meeting January 18, 1938.

British Museum, which reduced it to the proportions of a well-defined flood event which brought disaster to the inhabitants of the Euphrates Valley about 3400 B. C.

4.—During 1938 marked progress has been made in the collecting and publishing of records of historic floods in the United States. Noteworthy in this connection is the United States Geological Survey, *Water Supply Paper No. 836-A*, entitled "Stages and Flood Discharges of the Connecticut River at Hartford, Conn.," which contains a chronology of flood stages from 1639 to and including 1937. This report was issued before the occurrence of the September, 1938, flood. Chronologies of this kind are now available for about twenty rivers in the United States, and partial chronologies have been built up for as many more. Much remains to be accomplished. Fortunately, the number of engineers engaged or interested in this work has increased greatly. What has been accomplished to date has proved of value in several ways: First, by demonstrating the practicability of procuring significant data through intelligently conducted research; second, by showing that early flood happenings can be correlated with present-day gage records with sufficient accuracy for engineering purposes; and, finally, by extending the knowledge of the frequency of major flood occurrences, which after all is basic in all flood-control considerations.

Recent Publications

5.—Among the more important reports devoted to flood data that have come to the notice of this Committee, are the following:

The Ohio-Mississippi Valley Flood Disaster of 1937: Report of Relief Operations of the American Red Cross, Washington, D. C., 1938.—Contains noteworthy statistics on building losses, live stock losses, and number of families that received aid (food, clothing, and other maintenance) in eleven States.

Stages and Flood Discharges of the Connecticut River at Hartford, Connecticut, by H. B. Kinnison, M. Am. Soc. C. E., L. F. Conover and B. L. Bigwood, Assoc. Members, Am. Soc. C. E., U. S. Geological Survey, *Water Supply Paper No. 836-A*, 1938.—Noteworthy for the completeness of the record of historic floods which begins with 1639.

The Floods of March 1936, Part 2, Hudson River to Susquehanna River Region, U. S. Geological Survey, *Water Supply Paper No. 799.*

The Floods of March 1936, Part 3, Potomac, James, and Upper Ohio Rivers, U. S. Geological Survey, *Water Supply Paper No. 800.*

The Ohio and Mississippi River Floods of January–February 1937, by Bennett Swenson, in Monthly Weather Bureau Supplement, No. 37, 1938.—Contains complete meteorological data, accompanied by maps.

Report on Flood of March 2, 1938 in Los Angeles County Flood Control District, by M. F. Burke, Jun. Am. Soc. C. E., Hydraulic Engineer, under the direction of C. H. Howell, M. Am. Soc. C. E., Chief Engineer; 1938 (mimeographed).—A recital of factual and historic data, accompanied by maps.

Synoptic Analysis of the Southern California Flood of March 2, 1938, by Charles H. Pierce, M. Am. Soc. C. E., *Monthly Weather Review*, May, 1938.

Southern California Rain and Flood, February 27 to March 4, 1938, by Lawrence H. Daingerfield, *Monthly Weather Review*, May, 1938.

Analysis of Run-Off Characteristics, by Otto H. Meyer, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., November, 1938, pp. 1769–1786.—A discussion of improved methods of preparing unit hydrographs and distribution graphs.

Inventory of Unpublished Hydrologic Data, U. S. Geological Survey, *Water Supply Paper No. 837*, 1938.—Contains unpublished data which heretofore have not been readily available to engineers.

In preparation at the close of the year (1938) were the following:

Flood of Ohio and Mississippi Rivers, of January–February, 1937, U. S. Geological Survey, *Water Supply Paper No. 838*.

Major Texas Floods, 1935, U. S. Geological Survey, *Water Supply Paper No. 796-G*.

Floods in Canadian and Pecos River Basins in 1937, U. S. Geological Survey, *Water Supply Paper No. 842*.

Highest and Lowest Annual Stages of the Mississippi River and Principal Tributaries Referred to Present or Most Recent Gages to and Including 1937, published by Mississippi River Commission, Vicksburg, Miss.

6.—*Inventory of Flood Data.*—A central agency to act as a clearing house for flood data, such as has repeatedly been recommended by this Committee and the one preceding it, is now coming into existence. Steps were taken during 1938 by the U. S. Geological Survey, Water Resources Branch, to create a permanent file or depository of information on floods. According to advices received from the Chief Hydraulic Engineer of this Bureau it is planned to acquire as complete as possible information relating to flood-stages, discharges, and storm rainfall for every flood of importance. This information will include not only the floods which occurred during the periods covered by regular observations, but also the important early flood events which antedated the establishment of gages and systematic records. Although emphasis will be placed on basic hydrologic data, this will not be to the exclusion of more general types of flood information. Among the latter will be information on flood damage and on such flood-control projects, either contemplated or completed, as are deemed to be especially significant. In this new "Flood Inventory" all data are planned to be filed and catalogued on a drainage-area basis. Information of special value to the public will be made available for distribution in published form. This undertaking, which thus far has made only a modest beginning, but which promises to become one of the most important services rendered to the public at large and to the Engineering Profession in particular, has been initiated with funds made available by the Public Works Administration. The field work now being done includes the gathering of data pertaining to droughts. Although drought data are not within the purview of this Committee, it is on record nevertheless as having pointed out to the National Resources Committee in 1935 the advantages of carrying on jointly—that is, with the same funds and same personnel—research work pertaining to floods and droughts.

7.—*Meteorology of Flood-Producing Storms.*—The disastrous flood events of the past four years have given rise to demand from many quarters for informa-

tion as to how great floods are physically possible of occurrence. The question is a pertinent one in view of the fact that it cannot be answered by reference to old-time formulas or probability studies. Moreover, an answer to this question has become particularly urgent because of the fact that at no time in the history of the United States have so large a number of engineers been engaged in the design, construction, and operation of large flood-control projects as is now the case. A logical answer can be obtained only by going back to first principles, namely, (*a*) to determine the maximum storm rainfall that is possible of occurrence over any given water-shed through meteorological research; and (*b*) to determine what rates of run-off will result from such maximum rainfall. The advances that have been made of late in these two distinct fields of research are most promising. Especially encouraging is the work now being done by the United States Weather Bureau in furthering the theoretical as well as the practical working knowledge of the maximum rainfall intensities and distributions possible of occurrence in various sections of the United States. The objectives are clearly set forth in the words[2] of the late Willis R. Gregg, Chief of the Weather Bureau, as follows:

"A research project which is believed to be of very real significance is one that is now being conducted cooperatively with the Corps of Engineers to estimate spillway and waterway capacities. The procedure is to make a detailed study of major storms that have resulted in excessive precipitation and floods, transpose these storms within reasonable limits to other locations with respect to nearby river basins, take other factors such as topography, and snow cover, and a different temperature distribution, into consideration, and endeavor to determine what is the maximum runoff that is ever likely to occur. This is a direct application of the work that is being done in air mass analysis* * *. It is planned to extend this study to all principal river basins in the country."

The work is being done by a nucleus of about twenty specialists known as the Hydro-Meteorological Research Section of the River and Flood Division of that Bureau. This is indeed attacking the "maximum-possible flood" problem from a logical angle.

8.—The significance to the Engineering Profession of this new departure in estimating storm rainfall intensity and distribution, lies in the fact that this class of research normally is the function of the meteorologist rather than that of the hydrologist. Upon the latter, however, devolves the determination as to what flood run-off rates will result from the storm intensities determined by the meteorologist, and this will call for thorough field study of the water-shed with reference to run-off concentration. Both fields of endeavor, undoubtedly, are due for considerable development. The primary need, of course, is for more river-discharge and rainfall stations. The unit graph method, which has great possibilities, remains to be perfected. In this connection attention is invited to the paper by Otto H. Meyer, Assoc. M. Am. Soc. C. E., on "Analyses of Run-Off Characteristics,"[3] which presents improvements and suggestions for improvements in the use of unit graphs and distribution graphs which this Committee believes are destined to become of great assistance to engineers. In conjunction with the rainfall studies being made by the Hydro-Meteorological

[2] *Electrical Engineering*, October, 1938, p. 411.

[3] *Proceedings*, Am. Soc. C. E., November, 1938, pp. 1769–1786.

Research Section of the Weather Bureau these improved methods, it is hoped, will enable engineers in the near future to work out solutions practically free from the wide margin of uncertainty attached to present efforts. The Weather Bureau has no small task before it, but its findings, as they become extended over the entire country, will lay a foundation of inestimable value. This Committee deems it highly important, therefore, that the River and Flood Division of the Weather Bureau be furnished ample allotments for carrying on this basic work.

9.—Statistical Methods.—A word of caution is needed with reference to placing undue confidence in statistical methods for determining the frequency of occurrence of floods of various magnitudes. The tendency in some quarters has been to overrate their dependability. Some of the limitations inherent in these methods are as follows:

(*a*) The fundamental error involved in all frequency or probability studies is the arbitrary assumption that the data utilized are of a homogeneous character. This is equivalent to stating that, for any drainage basin, floods are floods. Actually, the floods in any drainage basin in the United States may be divided into distinct species produced by wholly dissimilar meteorological and often seasonal conditions. In the days when flood records were quite short, the meagerness of the data justified this assumption. To-day, this procedure no longer is permissible on water-sheds having records of 50 to 60 yr duration. It is advisable, in such cases, to sort flood events according to their respective origins and consider each type in a separate frequency study. For instance, floods produced by general winter rains, by storm rainfall induced by tropical hurricanes, by cloudbursts, by rapid snow melting plus rain, are in effect distinct types produced by wholly unrelated phenomena. These floods merit being dealt with separately. In different regions of the United States the number and nature of these types differ appreciably.

(*b*) Few of these methods admit of taking into consideration the important but isolated historic flood events preceding the period of regular observations. This is a serious shortcoming, more especially in these times when much information is being made available concerning early floods. Among the latter are some of the greatest floods in history, which must be given proper weight if the analysis is to be of real value.

(*c*) Purely statistical methods of dealing with observations of the kind under discussion permit of no latitude in assigning different weights to the observations used. This means that judgment is excluded and blind mathematics prevail. Unquestionably, this is the most indefensible shortcoming of applying the method of least squares to natural phenomena of so heterogeneous a character and differing so greatly in relative accuracy and in variability of occurrence in point of time as is the case with observations on floods. As a practical solution to this difficulty the use of graphical methods is here recommended. These methods enable the engineer to assign weights, through the use of symbols, and render an intelligent appraisal of the platted data possible.

10.—Progressive Distortion of Run-Off Conditions.—The shortness of flood records in the United States has been a matter of frequent comment. This

situation is not due for marked improvement. Increasing occupancy of the land, constant changes in vegetal cover in nearly every water-shed, with improved drainage, flood-water storage, irrigation uses, power development, and navigation improvements, combine to alter run-off rates. The annual hydrographs for many stream-flow stations in the United States are suffering distortion, both as to low-water and high-water stages and discharges. On some streams that have been canalized effectively over great distances, the low-water flow has lost all significance from the hydrologic point of view. On other streams, discharge must be measured over dams or through turbines and spillways. Storage works in some water-sheds have reduced flood flow to such an extent as to render estimates of true storm run-off a burdensome task. The number of streams affected is increasing rapidly each year. Although in most cases the total annual run-off has not been altered materially, marked changes are resulting in some instances due to changed temperature, evaporation, and infiltration conditions.

11.—The conclusion, therefore, is inescapable that, although stream-flow and stage records may gain in length as the years roll by, the records of the future will progressively lose in value as regards comparability with those of the past. This situation is certain to affect statistical flood studies unfavorably.

12.—Operation of Flood Control Works.—Now that a substantial foundation has been laid as regards the obtaining and publishing of flood data, this Committee considers it a duty to emphasize the importance of making the fullest possible use of the data available. This applies not only to the Civil Engineer, but to the Agricultural Engineer, the Forester, the City Planner, and to local authorities in areas where important flood problems exist. Probably one of the most important uses of flood data affects the operation of the many flood-control reservoirs and allied structures now being built throughout the United States. This phase of work should be under the direction of engineers having broad knowledge and training in hydrology, in the design, construction, and operation of such structures, and in the problems involved at multiple-use reservoirs. The succession of errors in operation which have been witnessed during the past few years at newly constructed flood-control works is at least in part due to placing this work in charge of men who have not had the proper qualifications. The disturbing fact in this connection is that few engineers have had occasion to become identified with the operating problems incident to such works. This Committee considers it of the utmost importance that public confidence in flood protection should not be permitted to be shaken by ineffective operation of structures correctly designed and built at great cost.

13.—Flood Damage.—This is an important phase of flood data, next in importance to flood records as regards being basic to any considerations affecting flood protection. This Committee wishes to reiterate its Recommendation No. 5 submitted in its Progress Report for 1935[4] to the effect that research is needed to determine the true relation between flood damage and the justifiable cost of control works. Research of this kind must be unbiased and fearless if it is to produce any worth-while results. Misconceptions appear to exist as regards what properly constitutes damage attributable to floods. Consider

[4] *Proceedings*, Am. Soc. C. E., February, 1936, p. 205.

the failure of a dam during a flood and its subsequent replacement. Correctly built, the dam would not have failed, for in essence a dam must be proof against destruction by water, much as a seaworthy ship should be proof against destruction by storms at sea. If it is not, the damage is chargeable to human error. This may take many aspects, such as: False economy, inadequate spillway provision, insecure foundations, neglectful maintenance, etc. What applies to dams applies equally to other types of structures. During the great flood of 1937 in the Ohio River no bridges across that river failed. They were built to withstand floods, and they stood. Had there been bridge failures on the Ohio, their loss or replacement cost would customarily have been classed as flood damage, and would have helped to swell the magnitude of the disaster and the argument for flood control. Whatever may have been customary, this Committee takes the position that the time is ripe for differentiating between types of damage. It questions whether all so-called flood damage affords a true basis for flood-control economics. A tentative classification of damage that does not appear to be properly chargeable to floods is as follows:

Class I

Structures Which Public Necessity Demands Shall Be Built in or Across Rivers and Must Be Proof Against the Action of Flood Waters.—(1) Dams; (2) bridges; (3) irrigation canal head-works; (4) hydro-electric plants; (5) navigation terminals; (6) navigation locks; (7) cable and pipe-line crossings; and (8) power transmission crossings. Only reasonable repairs and replacements incident to floods will be considered as constituting flood damage.

Class II

Structures Which Public Necessity Demands Shall Be Built in or Across Flood-Plains or on River Banks, and the Renewal or Rebuilding of Which, in Case of Damage by Flood Water, Is Economically Preferable to Building Structures not Destructible by Floods.—(1) Railroad embankments; (2) highway embankments; (3) bridge approaches; (4) revetment; (5) channel contraction works; and (6) ferries. How much of such reconstruction cost is properly chargeable to floods appears to be a moot question.

Class III

Buildings or Structures Which Public Necessity Demands Shall Be Kept Outside Flood Zones, or so Walled in as To Be Flood-Proof.—(1) Hospitals; (2) schools; (3) jails and penitentiaries; (4) insane asylums; (5) municipal water supply filtration plants and reservoirs; (6) telephone exchanges; (7) telegraph offices; and (8) post-offices.

14.—Such analyses as have been made of flood damage to agricultural interests tend to show that ordinary crop damage (excluding orchards, high-priced crops, and merchantable timber stands) rarely warrants the construction of costly protective works. Studies made in certain low areas of the Middle West indicate that the market value of farm land subject to overflow, rises and falls synchronously with that of farm land out of reach of flood waters. Market values appear to be regulated not by occasional losses from floods, or from

droughts, tornadoes, frost, glutted market, etc., but by more general economic conditions. The indications are that farm values rise during a succession of good years and decline during a succession of poor years. What constitutes a good year or a poor year depends as much upon the price received for the crop as upon the quantity raised. If these findings are generally true, then elimination of the flood menace from farm land assumes the nature of a quite incidental and much over-rated benefit not worthy of costly flood-control measures. These aspects are worthy of dispassionate study. It is quite possible that the plan followed in many localities, of protecting low lands by small inexpensive dikes to exclude the lesser but more frequent floods and to permit overflow by major floods to take place at rare intervals, is the logical solution.

15.—Flood damage to industrial property also invites dispassionate study. The situation has two distinct aspects: One is exemplified by the story of the Boy Scout who, having been taught to pitch his tent on high ground out of the path of flood waters, was perturbed upon return from camp to find his father building a factory on low ground subject to frequent overflow. Inquiring naively why, he was told that factories require large areas and that low ground was cheap; if floods should cause trouble, flood control could be procured without cost to his father. The other aspect is exemplified by the case of an industrial factory on the banks of the Lehigh River. After experiencing repeated flood damage the company removed its entire plant, about 25 yr ago, to high ground, and has been free from damage by floods ever since.

16.—Collateral Matter.—In submitting this report, the Committee realizes that the matter discussed in Paragraph 12 relative to the operation of flood-control works, and to a lesser extent in Paragraphs 13, 14, and 15, in which flood damage is considered, may be regarded as somewhat beyond the scope of its duties. These two problems arose repeatedly in the course of its studies and deliberations, and were found to be closely linked with the practical use and interpretation of data pertaining to floods and, therefore, basic in any considerations pertaining to flood control. It was the consensus of opinion of the Committee that it would be remiss in its duty if it did not call attention to the grave responsibilities confronting the Engineering Profession in the matter of flood-control operation, and to the need for analysis as to what properly constitutes flood damage.

Acknowledgments.—The Committee expresses its appreciation of the many helpful discussions of its reports that have appeared in the *Proceedings* of the Society.

Respectfully submitted:

Gerard H. Matthes, *Chairman*
Robert F. Ewald,
Frederick H. Fowler,
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December 15, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PRELIMINARY DESIGN OF SUSPENSION BRIDGES

Discussion

BY MESSRS. O. H. AMMANN, AND LEON BLOG

O. H. AMMANN,[33] M. AM. SOC. C. E. (by letter).[33a]—The authors have made a useful contribution to the design of stiffened suspension bridges. The principal merit of their method of analysis lies in the fact that it aids the student more clearly to visualize the behavior of the unstiffened cables and the effect thereon of the stiffening trusses and other factors, and to evaluate the economic value of stiffening trusses of various degrees of rigidity.

The paper does not attempt to go beyond the mathematical analysis as an aid to preliminary design studies; but in their conclusions the authors indicate that there are a number of problems in the design of stiffening trusses which require clarification. They may be summarized in the two important questions: What is the appropriate degree of rigidity? and how is it to be obtained structurally?

Judging from the widely varying degrees of rigidity of stiffening trusses, even in recently built suspension bridges, it would seem, indeed, that these questions offer a fruitful field for further study with a view to establishing a more uniform, and possibly improved, practice. However, a comprehensive study of the problem leads to the conclusion that both the criteria for rigidity and the factors involved in the design of an adequate and economical stiffening system are so complex, and to some extent necessarily matters of individual judgment, that an attempt to devise formulas or rules of design on a theoretical basis appears fruitless.

The problem is peculiar to suspension bridges because, contrary to its effect in truss types, rigidity is secured by increased expense and must be kept to a minimum. This, in fact, becomes one of the major economic factors in suspension bridges of long span.

NOTE.—The paper by Shortridge Hardesty and Harold E. Wessman, Members, Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. A. W. Fischer, and Jacob Karol; June, 1938, by Messrs. Glenn B. Woodruff and Norman C. Raab; September, 1938, by Messrs. Hardy Cross, and A. A. Eremin; October, 1938, by Messrs. A. Fraser Rose and William A. Rose; and November, 1938, by Leon S. Moisseiff, M. Am. Soc. C. E.

[33] Director of Eng., The Port of New York Authority, New York, N. Y.

[33a] Received by the Secretary November 16, 1938.

The very object of rigidity in a bridge is complex. In smaller and lighter structures the principal object is rigidity against vibrations or oscillations which may be detrimental to the structure as well as objectionable to traffic. No valid formula has ever been devised on theoretical grounds as a measure for the required degree of rigidity in this respect. Experience has set up very crude and purely empirical criteria in the form of limiting deflections.

As the bridge increases in span and weight this object becomes less and less important, except in the design of the floor system. In fact, flexibility of the structure as a whole becomes advantageous, because it tends to reduce local impact effects and vibrations. The principal purpose of limiting deflections and other deformations of the main carrying system in long spans is to avoid excessive gradients, and to prevent deformations that may result in excessive local bending, distortions, or transverse tilting of the floor structure.

Limiting grades may be important in bridges carrying rail traffic, but for modern highway traffic even that object loses significance, because the grade changes that may be actually produced in a well designed structure are not sufficient to form a serious impediment to traffic.

The effect of the deflections upon the floor depends so much upon the width, depth, and general arrangement of the floor structure as to require careful study in each individual case.

The authors state that present tendencies are to make stiffening trusses more flexible. This is undoubtedly due to the recognition of the fact that with increasing magnitude of bridges, and particularly those serving modern highway traffic, rigidity, as measured by deflections of the structure as a whole, ceases to be a virtue, and that flexibility offers material economic advantages, besides aiding the designer to produce more graceful structures.

This recognition is due partly to the abandonment of the so-called elastic theory which ignored the influence of the dead weight of the structure upon the proportioning of the stiffening system and thus, with increasing magnitude of span, led to more erroneous and wasteful design. The general rule, found in many former treatises and textbooks on suspension bridges—that the most appropriate depth of stiffening trusses, irrespective of span length, character of moving load, or dead weight of suspended structure, was between $\frac{1}{40}$ and $\frac{1}{60}$ of the span—was based upon that erroneous theory.

The writer became aware of the economic significance of flexible trusses in his preliminary studies for the George Washington Bridge. In the attempt to determine the proper proportions of the stiffening trusses for a span of such unprecedented length, he resorted to a careful study by analysis, as well as by model, of the behavior of the unstiffened cables, in respect to deflections, grade changes, and angular distortions under various load conditions.

These studies convinced him that this bridge, which was to be designed principally for highway traffic with the possibility of later addition of suburban transit trains, would have ample rigidity to accommodate highway traffic without any stiffening system whatsoever—that is, without any restraint upon grade changes and deflections of the unstiffened cables—but that, even for rapid transit trains, although the unstiffened system as a whole would be amply

rigid in so far as deflections and grade changes are concerned, comparatively flexible trusses between the two decks, with a depth ratio of only $\frac{1}{120}$, were advisable to secure local rigidity and avoid excessive local distortions of the floor.

The bridge was actually designed in accordance with these conclusions, and to date (1938) it has only a single completely unstiffened deck. This was a radical departure from conventional practice,[34] but experience has corroborated the validity of the conclusions.

The Golden Gate Bridge, at San Francisco, Calif., which has since been completed with a span of 4 200 ft and carries only highway traffic on a single deck, was designed with trusses 25 ft deep, which is only $\frac{1}{168}$ of the center span. Thus far, it also has proved amply rigid. It is the writer's present opinion that even shallower trusses or solid plate girders would be just as adequate for this great span.

A further radical step in the direction of greater flexibility of stiffening trusses was taken in the design of the Bronx-Whitestone Bridge across the East River in New York, N. Y. It has a center span of 2 300 ft, side spans of 735 ft, and is designed to carry only highway traffic on a single deck 75 ft wide. It is stiffened by plate girders of a depth of 11 ft, which is only $\frac{1}{210}$ of the span length. The moment of inertia of the girders is only 2 600 in.^{2}–ft^2 in the center span, and 2 140 in the side spans. These values are practically the same as the corresponding ones of the stiffening girders (10 ft deep) used in the Maumee River Bridge in Toledo, Ohio, which has a center span of only 778 ft, and side spans of 230 ft.

The Bronx-Whitestone Bridge still requires the test of actual operation. If it proves to be amply rigid, as the writer does not doubt, it should set a precedent for spans of that magnitude and similar proportions. It should also indicate that some of the bridges of shorter span might well have been designed with shallower, more economical, stiffening trusses or girders, particularly such structures in which the deflections of the unstiffened cables are reasonably restrained by the compound effect of other influencing factors, such as relatively wide and heavy floor structure, shallow cable sag, and short side spans.

Suspension bridges for heavy modern highway traffic are a comparatively recent development. They will never be common structures and each case will call for and merit most careful study. In view of the complexity of the factors which influence the design of the stiffening system, the trial method of approaching a final design, together with judgment based upon experience, will be found as serviceable as and less likely to lead to erroneous practice than "cut and dried" formulas or rules of design.

LEON BLOG,[35] ASSOC. M. AM. SOC. C. E. (by letter).[35a]—A method for the preliminary design of two-hinged stiffening trusses of cable suspension

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[34] *Transactions*, Am. Soc. C. E., Vol. 97 (1933), p. 43.

[35] Asst. Chf. Structural Engr., Div. of Bridges and Structures, City of Los Angeles, Los Angeles, Calif.

[35a] Received by the Secretary November 29, 1938.

bridges is presented in this paper. The object of developing a preliminary method for proportioning any structure is the rapid consummation of the final design. The use of the method must assure the designer that the proportions selected for the application of the preliminary method will result in the ideal design when used in a more accurate method. The preliminary method presented by the authors enables the designer to arrive, rapidly and understandingly, at deflections and moments, thereby yielding an idea as to the combined stiffness of the cable and stiffening truss, the effect of the live load on the total cable tension, and the unit stresses in the truss selected.

The writer has investigated the possibility of further reducing the total time for the preliminary design required by ignoring the width of the top of the tower. This tower width is a factor in the authors' method and, of necessity, considerable time must be spent in investigating for a rational width of tower top in order to arrive at L', the short cable section over the saddle. The horizontal component of the live load tension as well as that of temperature is a function of $I \div I_1$. The horizontal components are first converted into tensions along the cable and these tensions, in turn, into vertical components acting on the tower. The tower can then be designed. Thus, the final tower design cannot proceed without satisfactory stiffening trusses. If a suitable method of determining such trusses can be developed independently of a predetermined tower-top width, considerable time will be saved.

Accordingly, the writer has explored the degree of accuracy to be obtained by using the authors' method with all steps involving L' deleted from their solutions. To do this, the cable curves were produced backward to the center lines of the towers, new sags, cable lengths, and horizontal spans were computed and then used in the authors' solutions with Operations 6(c), 7(c), and 8(c), in Table 2(a), eliminated.

Design constants in Table 1(a) are compared with the results of the writer's simplification in Table 4.

TABLE 4.—COMPARISON OF DESIGN CONSTANTS

Method	Length of Span, in Feet		Length of Cable, in Feet		Sag at Center of Span, in Feet	
	Main span, l	Side span, l_1	Main span, L	Side span, L_1	Main span, f	Side span, f_1
Table 1(a), Column (2)	1 358	660.7	1 394.00	685	133.70	31.600
Writer's simplification	1 380	671.7	1 416.82	696	138.07	32.337

The authors have observed that the horizontal projection of L' practically equals the true value of L' which is supported by noting that $L = 1\,416.82$ ft $= 1\,394 + 22.82$ ft, the additional 0.82 ft arising from the fact that the main and side-span cables intersect at the center line of the tower in a cusp, the sum of whose sides is greater than $L' = 22$ ft. Note that the side-span cable has been increased 11 ft to 696 ft, from 685 ft. In short, the total length of the bridge cable is practically unchanged. Dropping the 0.82 ft and using

1 416.00 ft did not affect the writer's solutions. The writer's values in Table 4 were used in solving Tables 2(*a*), 2(*b*), 2(*c*), and 2(*e*). The answers obtained by the authors and by the writer's simplification of their method are compared in Table 5.

The writer did not solve the cases of Tables 2(*d*) and 2(*f*) because no step-by-step check was available. However, the nature of the computations in the authors' method indicates that results close to those of the authors for these unsolved cases would result by using the writer's simplification.

Table 5 shows that the maximum deviation for moment for any case studied was about 2% less than the authors' result and occurs in Table 2(*b*). Assuming that the stiffening trusses being investigated are actually working up to a maximum allowable fiber stress of 18 000 lb per sq in., the writer's result

TABLE 5.—COMPARISON OF MAXIMUM MOMENTS AND CORRESPONDING DEFLECTIONS

Item No.	Method	Quarter-point, Main Span* (Table 2(*a*))		Center, Main Span* (Table 2(*b*))		Quarter-point, Main Span† (Table 2(*c*))		Center, Side Span* (Table 2(*e*))	
		Moments, in foot-kips	Deflection, in feet	Moments, in foot-kips	Deflection, in feet	Moments, in foot-kips	Deflection, in feet	Moments, in foot-kips	Deflection, in feet
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Authors'.....	28 500	4.58	23 600	4.90	23 300	2.93	43 400	3.98
2	Writer's......	28 545	4.68	23 200	4.98	23 160	3.08	43 700	4.13
3	Ratio‡.......	1.001	1.021	0.983	1.017	0.995	1.050	1.007	1.037

* Maximum positive moments; deflection is down. † Maximum negative moment; deflection is up. ‡ $\frac{\text{Item No. 2}}{\text{Item No. 1}}$.

yields $0.02 \times 18\,000 = 360$ lb per sq in. less, or the stress in the chords would be 17 640 lb per sq in. Inspection of Table 5 shows that the variations for other cases in Table 2 are not large. Since the essence of the simplification being demonstrated is the use of longer stiffening trusses than those proposed, with the incidental greater loads, there is never any danger of adopting trusses that will be too weak.

Table 5, Column (9), further shows that the maximum deviation between the two methods for any downward deflection is plus 4% and equals about 2 in.; and, Table 5, Column (7), shows the deviation for the upward deflection to be 5% and also equals about 2 in. Even for so stiff a structure as the modern Triborough Bridge (New York City), a 2-in. variation in accuracy between two preliminary methods is not serious.

The results obtained by the writer's simplification are evidently similar to those of the authors. No preliminary tower design is required. Time is saved. One more trenchant tool has been added to the engineer's kit by the authors. Once the underlying formulas are understood and accepted, only the simplest mathematics is required. Visualization of the otherwise intricate interaction of all parts of the hybrid assembly has been made easy.

"Preliminary Design of Suspension Bridges" will take its place beside "Analysis of Continuous Frames by Distributing Fixed-End Moments"[36] as a drudgery reducer in structural analysis.

Corrections for *Transactions:* In the denominator of Equation (40*a*) change "l^2" to read "l_1^2"; in Table 2(*c*), "Explanation" to Step 10, change "$H_a = 1\,000$ kips" to read "$H_a = 2\,250$ kips"; and, in Table 2, "Computation," make four corrections as follows: Table 2(*a*), Step 10(*a*), change "$\times$" to "$+$" in the denominator; Table 2(*b*), Step 1, change "133.7×1.78" to read "$133.7 + 1.78$"; Table 2(*c*), Step 4, change "10^8" to "10^6"; and, in Table 2(*e*), Step 1, change "$-$" to "$\div$."

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[36] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

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DISCUSSIONS

STRUCTURAL BEHAVIOR OF BATTLE-DECK FLOOR SYSTEMS

Discussion

By Inge Lyse, M. Am. Soc. C. E., and Ingvald E. Madsen, Jun. Am. Soc. C. E.

Inge Lyse,[11] M. Am. Soc. C. E., and Ingvald E. Madsen,[12] Jun. Am. Soc. C. E. (by letter).[12a]—An experimental solution of a complicated structural problem must of necessity be limited within the available resources. For a problem as complicated as that of the battle-deck floor system the test program had to be greatly limited in scope. Mr. Jones describes and justifies this limitation so well that it is unnecessary for the writers to elaborate on it. Mr. Jones thus answers some of the questions and criticism raised by Messrs. Hill and Moore, and since the writers subscribe in full to the discussion by Mr. Jones they will concern themselves only with other facts of the discussion by Messrs. Hill and Moore.

In discussing the bending moments and the load-distribution factors Messrs. Hill and Moore state that Table 1 gives computed center moments for Stringers B and D on either side of the loaded stringer of the full-sized floor, which are greater than would have been obtained had the loads computed for these stringers been concentrated at the center of the span. According to Table 1 Stringer B carried 4 850 lb; the span length is 16.75 ft; so that the center moment becomes $M_c = \frac{4\,850 \times 16.75 \times 12}{4} = 244\,000$ lb-in. against 250 000 lb-in. given in the table. It seems that this is well within experimental errors.

The width of plate in T-beam action is also questioned and reference is given to theoretical analyses indicating that for the full-sized floor panel an effective width of 45 in. could be assumed when spacing of the stringers was

Note.—The paper by Inge Lyse, M. Am. Soc. C. E., and Ingvald E. Madsen, Jun. Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1938, by Messrs. H. N. Hill, and R. L. Moore; and September, 1938, by Jonathan Jones, M. Am. Soc. C. E.

[11] Prof. of Reinforced Concrete and Solid Bridges, Inst. of Technology, Trondheim, Norway.

[12] Associate in Civ. Eng., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

[12a] Received by the Secretary October 26, 1938.

only 24 in. It seems to the writers that the maximum effective width could not very well exceed the stringer spacing. It should also be noted that the T-beam analysis, referred to in the discussion, is based on a single T-beam with very wide flanges, which is not the same case as the series of parallel beams in the battle-deck floor. Furthermore, the observed tension in the stringers agreed very well with the width of plate given in Table 3.

Messrs. Hill and Moore compare the results of the battle-deck steel floor with aluminum plates on channel sections. It seems that this is an extrapolation of the writers' experimental results beyond reasonable limits. The differences obtained, therefore, are what might be expected when materials and sections that differ so widely are used. It cannot be emphasized too often that experimental results of complicated structures must not be generalized to apply far outside the range covered in the experiments.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

WATER-HAMMER PRESSURES IN COMPOUND AND BRANCHED PIPES

Discussion

By Robert W. Angus, Esq.

Robert W. Angus,[33] Esq. (by letter).[33a]—The discussion by Professor DeJuhasz shows that the matter dealt with is of broad application. The writer agrees with the suggestion that the results become much clearer if represented by a solid instead of a plane diagram, and the elegant models made by Professor DeJuhasz[34] illustrate the meaning of the diagrams well. For problems met with in water-works and hydraulic systems, however, it is only necessary to determine the magnitude and location of the most dangerous pressure by the use of the diagrams shown.

It must be admitted that the method of dealing with friction is open to some criticism, but as the number of points of concentration of the friction is increased, the assumed pipe approaches the actual case more and more closely. If the number of points was infinite, the solution would be an exact one from the friction standpoint. Therefore, it becomes largely a question as to the number of points to concentration that must be used to avoid serious errors in the results, and judgment thus comes into the question. As the diagrams may be drawn very quickly, even the inexperienced person can try the same problem with one, two, three, or more, points of concentration, and in that way the minimum number is soon found; but the writer has observed that it is rarely necessary to assume more than two points, and very often one is sufficient.

In Fig. 20 the solution is offered for the case of two points of concentration where the resistance, $A_0 D_3$, is a little greater than 10% of the static head; but it would have been just as easy to solve the problem had $A_0 D_3$ been taken, say, at 95% of the static head, as would be the case in many water-works

Note.—The paper by Robert Angus, Esq., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1938, by Messrs. K. J. DeJuhasz, and Harold A. Thomas; and September, 1938, by Messrs. F. Knapp, Martin A. Mason, Pierre F. Danel and Antoine Craya, and A. A. Kalinske.

[33] Prof. of Mech. Eng., Univ. of Toronto, Toronto, Ont., Canada.

[33a] Received by the Secretary November 17, 1938.

[34] "Hydraulic Phenomena in Fuel Injection Systems for Diesel Engines," by K. J. DeJuhasz, *Transactions*, A. S. M. E., November, 1937, p. 669.

problems. Usually, sufficient accuracy is obtainable by assuming all the loss concentrated at the pipe entry and the corresponding diagram is easily deduced (see Fig. 25).[35]

Several of the discussers have dealt with friction, suggesting different ways of allowing for it, and some of these solutions may approach closer to the correct results than any yet suggested. There was no attempt in the paper to minimize the effect of friction, but it makes the diagrams more difficult to follow than if it were omitted; and the very small size of the diagrams suggested to the writer that it was best to omit it in most cases. The writer includes it in dealing with actual problems, and believes that, on the whole, the errors due to this phase of the work are much less than those due to uncertainties in the value of V_w and the exact effects caused by the turbine governor or other source of disturbance.

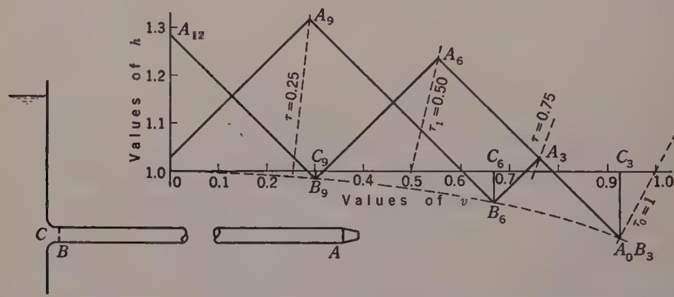

FIG. 25.—EFFECT OF FRICTION FOR STRAIGHT-LINE CLOSURE IN 12 SECONDS; $\rho' = 0.5$; $\frac{L}{V_w} = 3$ SECONDS

The analytical solution given by Professor Thomas is most instructive, and may also be derived from the writer's diagrams by writing down in succession the equations to the various lines on the diagrams. Since the slope of each line is known, and also its starting point, it is not difficult to solve for each point without the use of the drafting-board. The analytical solution forms a valuable check on the graphical one. There is more danger of error in the former than in the latter method, however, and it appears easier to detect mistakes on the diagrams than on the tabulated set of data. Mistakes may easily be made on the diagrams by drawing lines at the wrong slope, but once the correct slopes are established they are transferred very easily from place to place.

The writer has found that it is essential to be very careful with the slopes of the lines; otherwise they may become mixed when they are nearly parallel, and for this reason he used a separate diagram for each pipe, as in Fig. 14, etc., of the paper, in preference to the combined diagram, Fig. 23, shown by Professor Thomas. With the particular data of Fig. 23, that diagram is quite clear, but in many cases the lines become very much entangled and then the separate diagrams are best. The writer always puts all axes of v or V in the

[35] "Water Hammer in Pipes, Including Those Supplied by Centrifugal Pumps: Graphical Treatment," by R. W. Angus, *Proceedings*, Inst. M. E., London, England, Vol. 136 (1937), Fig. 23*a*, p. 288. (Slightly revised.)

same horizontal line when working problems, so as to reduce the labor somewhat. Many geometrical shortcuts will suggest themselves, of which the one shown by Professor Thomas is ingenious.[35a] Scales must be arranged so that intersections are not too acute, and frequently it is desirable to use two vertical scales on the same drawing; in one case the writer found it convenient to use a scale for H of 10 ft = 1 in. up to H_0 = 100 ft, and 50 ft = 1 in. for values of H_0 greater than 100 ft; the engineer quickly adjusts himself to these solutions.

The points raised by Mr. Mason, Professor Danel, and Mr. Craya touch on several important matters. Undoubtedly, in principle, velocity head should be taken into account, as the writer has stated, and as is further emphasized by this discussion. It may be allowed for in various ways; thus, in Fig. 20, it has been treated similarly to friction by adding the velocity head to the friction loss, which has lengthened such lines as $B_3 B_3'$. In general, velocity head is relatively small and has very little influence. The modification shown in Fig. 24 is neat and ingenious and does not seriously lengthen the work.

Those who read the analytical method of computing water-hammer will appreciate the demonstration given by Messrs. Mason, Danel, and Craya of the derivation of the formulas for the transmission and reflection coefficients, which are difficult to understand and are well illustrated in the diagrams. The graphical construction includes them without appearing to do so. The derivation given shows the connection between the two methods of treatment and is a distinct addition to the literature on the subject.

The writer is pleased to find the desire, on the part of so many workers in the subject, to compare the analytical and the graphical methods. Jaeger[6] has been one of the exponents of the former method and Equation (84*b*) shows the connection of one of his important coefficients with the corresponding lines on the graphical diagram. Undoubtedly, the two methods must go side by side, recourse being had to parts of one and parts of the other where necessary; and the writer realizes that his own statement that in some cases analytical solutions were almost impossible, is capable of a much wider interpretation than was intended.

The three writers have called attention to a practical difficulty in solving water-hammer problems by any method due to the lack of exact knowledge of the pressure-velocity-time relations for the disturbing valve or gate; the solution cannot be more accurate than the data in this respect.

The discussion written by Mr. Knapp calls attention to the solution of the turbine and draft-tube problem illustrated by Fig. 17. Naturally, the turbine would vary somewhat in speed during gate closure and this would add to the disturbances entering the problem, but if all these features were discussed there would be ample material for a complete paper on that one problem. The writer was trying to illustrate the many different types of problems that could be solved, and Fig. 17 shows one in which he had been interested for some time.

The writer cannot conceive of a competent engineer deliberately designing

[35a] Corrections for *Transactions:* In Table 1, change values in the column headed B_A4 as follows: Item No. 1, 9.14; Item No. 3, 126.0; Item No. 10, 6.44; Item No. 12, 261.0; and, correct the remainder of the table correspondingly. Footnote numbers have been changed from 7 to 17, inclusive, to read 6 to 16 inclusive.

[6] "Theorie General du Coup de Belier," by Charles Jaeger, Dunod, Paris, 1933.

a plant with the characteristics at Point D of Fig. 15, and then correcting it with a surge tank as in Fig. 16; nor has he suggested making the plant stable by means of the surge tank. Fig. 15 represents a type of plant that is common —that is, one in which a single supply pipe, $C\ B$, delivers water to two penstocks terminating in gates or nozzles used for turbines. In Fig. 15(b) the writer selected an arbitrary variation of h with v for the outlet at Point D and then investigated what happened. The inference from the investigation, clearly, is that such an arbitrary variation at Point D is incorrect; there may be other ways of determining this fact but the construction shown in the paper is a simple one.

The writer then added one more accessory to the plant by examining a surge tank located at Point B with the results shown in Fig. 16; but whether or not that is a desirable method has not been discussed. The tank certainly reduces the pressure swings and makes the system practicable "as far as the investigation has gone," and is "a further example of how these difficult problems may be solved simply and accurately, on the drafting board."

In regard to the derivation of Equations (10) the writer feels that he could scarcely have been expected to read the paper to which Mr. Knapp refers.[22] There are many methods of deriving these equations, the one adopted being sound and as reliable as any. This method appeals to some, but others may prefer a different one; and it makes little difference in the final outcome so long as there is confidence in the correctness of the equations.

The statements by Mr. Knapp that the writer worked "under assumptions that are fundamentally incorrect" and has obtained "results in accordance with these assumptions" is rather broad and, with a single qualification, to be discussed subsequently, is tantamount to a denial of Newton's Second Law. Before studying the paper in which Mr. Knapp calls "attention[18] to the incorrect solution of the problem under discussion," the reader would do well to analyze, carefully, the conditions imposed by the writer in solving the problem of Fig. 18 and the similar problem[21] reported by the writer in 1937. In referring to Fig. 18, the present paper states that "a vacuum valve is installed at Point B in such a manner as to prevent the pressure falling to that extent" (namely, below atmospheric pressure), and further, "the admission of air and its subsequent release at Point B maintains atmospheric pressure there until the columns re-unite." A similar statement is made for the 1937 paper to which Mr. Knapp refers[21] and in both cases it is assumed (although only stated in the latter), that Pipe $C\ B$ rises somewhat in the vicinity of Point B so that the surface of the water must be horizontal while the columns are divided.

Under the first assumption, there could be no connection whatever between Columns $A\ B'$ and $B''\ C$ from the time they part until they re-unite; and, both may be treated as distinct columns, as has been done, the air at Point B being

[22] "O golpe de arieta: Theoria, confirmação experimental e applicaçoes praticas," by F. Knapp, *Boletin* da Inspectoria de Serviços Publiços, São Paulo, November, 1937.

[18] "Ueber eine allgemeine graphische Berechnungsmethode der Druckstoesse in Rohrleitungen," by F. Knapp, *Wasserkraft und Wasserwirtschaft*, 1935, p. 279; also, "Operation of Emergency Shut-Off Valves in Pipelines," by F. Knapp, *Transactions*, A. S. M. E., November, 1937.

[21] The same criticism applies to the author's paper, "Air Chambers and Valves in Relation to Waterhammer," *Transactions*, A. S. M. E., November, 1937.

always at atmospheric pressure. Mr. Knapp's first paper[18] refers to the case in which air is not admitted at Point B and where cavitation may occur with somewhat uncertain effects, in the present knowledge of this phenomenon. It is unfortunate that he has not given full details of his experiments[18] so that those interested might form an independent judgment on this subject.

The writer agrees with the criticism of the profile of Pipe $B\ C$ which should be always at least 66 ft below Surface C. The actual case suggesting this problem was of a pipe where the total rise in Line $B\ C$ occurred very close to the reservoir, in which case the solution is correct. It is only to this point that Mr. Knapp's sweeping criticism applies.

In his discussion, Mr. Kalinske thinks that the writer should have emphasized the fact that the method described is one for solving a set of simultaneous equations; but as Equations (23) and (26) make this quite clear, he had not thought it necessary to enlarge further on the fact. Instead of depending on an undue number of equations, a better grasp of the subject can be obtained from the broad variety of illustrations shown.

The writer agrees with Mr. Kalinske that with most engineers a clear conception of what happens in water-hammer is best obtained by first working a number of examples in a step-by-step process, and he uses this method with students as an introduction to the subject. He analyzed the "wave plan" described by R. Löwy,[36] and illustrated by Mr. Knapp,[13] but felt that it was not helpful enough to justify the time spent on it.

The writer expresses his thanks to all those who have been kind enough to contribute to this paper.

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[36] "Druckschwankungen in Druckrohrleitungen," by R. Löwy, pub. by Julius Springer, Berlin, 1928.

[13] "High Head Penstock Design," by A. W. K. Billings, M. Am. Soc. C. E., O. H. Dodkin, F. Knapp, and A. Santos, Jr., Assoc. M. Am. Soc. C. E., First Symposium on Waterhammer, 1933. Limited Special Edition; distributed by A. S. M. E.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DESIGN OF PILE FOUNDATIONS

Discussion

BY A. AGATZ, ESQ.

A. AGATZ,[28] ESQ. (by letter).[28a]—The formulas and principles of design given by the author are, in fact, applications, to special and simple cases, of the methods developed by Dr. Nökkentved.[5] It is fortunate that these applications have permitted the emphasis of certain fundamental rules and that Mr. Vetter has demonstrated the importance of the rotating moment, M.

The main disadvantage in the application of Dr. Nökkentved's general methods, or any special method derived therefrom, is that only stresses in pile foundations that have already been designed, may be determined. Except in the case of pile foundations having piles in two directions only, or having three piles only, it is not possible, by the method presented by the author, to design a foundation for which it is assured beforehand that all piles will receive approximately equal loads for a given exterior loading. Making use of the same general assumptions as those of Dr. Nökkentved and the author, it is possible, however, by using a somewhat different approach to design a foundation consisting of piles in three directions, in which the loading on each individual pile is predetermined for one, and even for two, different exterior loadings. If more than these two loadings must be resisted, it will be necessary to choose Point O at or near the points of intersection of the resultants of the various loadings so as to reduce the magnitude of the rotating moment.

Since a moment always produces a rotation about the elastic center, with resultant non-uniform pile loads, it is desirable to design the pile foundation in such a way that the most unfavorable exterior load produces translation without rotation. The elastic center of a pile system which satisfies this condition falls on the particular resultant, R. Let the resultant that produces a translation (without rotation) in a direction forming an unknown angle, η, with the vertical, be designated R_η; and, let the load on an arbitrary pile as a

NOTE.—The paper by C. P. Vetter, M. Am. Soc. C. E., was published in February, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. Hibbert M. Hill, and Odd Albert; June, 1938, by Messrs. August E. Niederhoff, A. A. Eremin, and Jacob Feld; and September, 1938, by John M. Coan, Jr., Jun. Am. Soc. C. E.

[28] Prof., Technische Hochschule, Berlin, Germany; tr. by C. P. Vetter, M. Am. Soc. C. E.

[28a] Received by the Secretary May 23, 1938.

[5] "Beregning af Pälevärker," by C. Nökkentved, Copenhagen, 1924.

result of the translation be P_n and the axial deformation of an arbitrary pile, be ΔL_n, then:

$$P_n = \frac{E_n\, A_n\, \Delta L_n}{L_n} \qquad (63)$$

in which L_n and A_n are the length and cross-section area, respectively, of the pile, and E_n is the elastic modulus of the pile material. If all pile heads are at the same level and all pile ends likewise at the same level, at a depth, d, below the heads (see Fig. 17):

$$P_n = \frac{E_n\, A_n}{d}\, \Delta L_n \cos \alpha_n \qquad (64)$$

in which d is the vertical projection of the pile length, and α_n is the angle of the pile axis with the vertical. Let it further be assumed that all piles are of the same cross-section area and the same material. Since, in the investigation

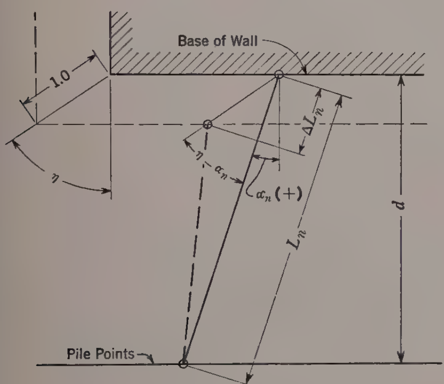

FIG. 17

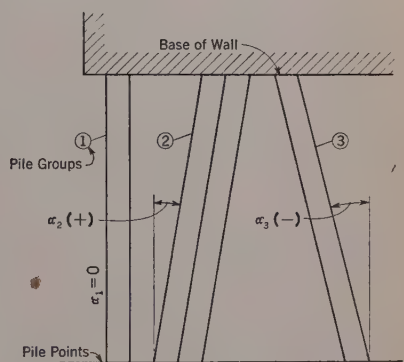

FIG. 18

that follows, it is the ratios between the pile loads in the various groups that are of primary interest, the factor, $\frac{E_n\, A_n}{d}$, then may be given the value unity, without affecting those ratios, so that:

$$P_n = \Delta L_n \cos \alpha_n \qquad (65)$$

If the translation in the direction, η, is also unity:

$$\Delta L_n = \cos(\eta - \alpha_n) \qquad (66)$$

and, it follows that,

$$P_n = \cos(\eta - \alpha_n) \cos \alpha_n \qquad (67)$$

If $\eta = 0$ (vertical translation), $P_n = \cos^2 \alpha_n$; and if $\eta = 90°$ (horizontal translation), $P_n = \sin \alpha_n \cos \alpha_n$. Let the loads in Pile Groups 1 and 3 (Fig. 18) be expressed as ratios, n and m, of the simultaneous pile loads in Group 2, all due to the same translation; thus, if $P_2 = \cos(\eta - \alpha_2) \cos \alpha_2$ (see Equation (67)): $\frac{P_1}{P_2} = n$; and, $\frac{P_3}{P_2} = m$.

Consequently,

$$P_1 = n\,P_2 = n\cos(\eta - \alpha_2)\cos\alpha_2 = \cos(\eta - \alpha_1)\cos\alpha_1 \ldots\ldots (68a)$$

and,

$$P_3 = m\,P_2 = m\cos(\eta - \alpha_2)\cos\alpha_2 = \cos(\eta - \alpha_3)\cos\alpha_3 \ldots\ldots (68b)$$

It will be assumed, furthermore, that the piles in Group 3 are in tension so that α_3 is negative, the numerical value in α_3 being designated $|\alpha_3|$. Equation (68*b*), then, is expressed as:

$$P_3 = \cos(\eta + |\alpha_3|)\cos|\alpha_3| \ldots\ldots (69)$$

Various relationships between α_1, α_2, α_3, η, m, and n may be derived from Equations (68*a*), (68*b*), and (69) as follows:

Case a.—Given, n, m, α_1, and α_2; let it be required to determine η and α_3. Then, from Equations (68*a*) and (68*b*),

$$\tan\eta = \frac{\cos^2\alpha_1 - n\cos^2\alpha_2}{n\cos\alpha_2\sin\alpha_2 - \cos\alpha_1\sin\alpha_1} \ldots\ldots (70a)$$

For $\alpha_1 = 0$, Equation (70*a*) becomes:

$$\tan\eta = \frac{1 - n\cos^2\alpha_2}{n\cos\alpha_2 - \sin\alpha_2} \ldots\ldots (70b)$$

and, similarly,

$$\cos 2\,\alpha_3 = \frac{q \pm \tan\eta\sqrt{1 - q^2 + \tan^2\eta}}{1 + \tan^2\eta} \ldots\ldots (70c)$$

in which,

$$q = 2\,m\,(\cos^2\alpha_2 + \tan\eta\sin\alpha_2\cos\alpha_2) - 1 \ldots\ldots (70d)$$

In Equations (70) m and α_3 may be substituted for n and α_1, respectively, to determine the values of η and α_1.

Case b.—Given, $\alpha_1 = 0$, α_2, α_3, and m; let it be required to determine η and n. Similarly to Equation (70*a*):

$$\tan\eta = \frac{\cos^2|\alpha_3| - m\cos^2\alpha_2}{\sin|\alpha_3|\cos|\alpha_3| + m\sin\alpha_2\cos\alpha_2} \ldots\ldots (71a)$$

and,

$$n = \frac{1}{\cos\alpha_2(\tan\eta\sin\alpha_2 + \cos\alpha_2)} \ldots\ldots (71b)$$

Case c.—Given, α_1, α_2, α_3, and η; let it be required to determine m and n. Then, proceeding as in Case (*b*),

$$n = \frac{\cos(\eta - \alpha_1)\cos\alpha_1}{\cos(\eta - \alpha_2)\cos\alpha_2} \ldots\ldots (72)$$

For $\alpha_1 = 0$, Equation (72) becomes:

$$n = \frac{\cos\eta}{\cos(\eta - \alpha_2)\cos\alpha_2} \ldots\ldots (73a)$$

and,

$$m = \frac{\cos(\eta + |\alpha_3|)\cos|\alpha_3|}{\cos(\eta - \alpha_2)\cos\alpha_2} \ldots\ldots (73b)$$

Equations (73) are of particular interest for the design of pile foundations, and numerical values of m and n for varying values of α_2, α_3, and η are given in Table 5.

TABLE 5.—VALUES OF m AND n FOR CORRESPONDING VALUES OF α_2, α_3, AND η ($\alpha_1 = 0°$ (SLOPE = 1 : ∞))

α_3			α_2							
Slope	Angle	η	SLOPE = 1 : 3; AND, ANGLE = 18° 28′		SLOPE = 1 : 4; AND, ANGLE = 14° 2′		SLOPE = 1 : 5; AND, ANGLE = 11° 19′		SLOPE = 1 : 6; AND, ANGLE = 9° 28′	
			m	n	m	n	m	n	m	n
1 : 3	18° 28′	75°	−0.110	+0.495	....	....	....	....	....	....
		80°	−0.309	+0.384	−0.353	+0.440	−0.391	+0.488	−0.449	+0.528
		85°	−0.585	+0.231	−0.699	+0.276	−0.804	+0.316	−0.884	+0.354
		90°	−1.000	+0.000	....	....	....	....	....	....
1 : 4	14° 2′	80°	−0.151	+0.384	−0.173	+0.440	−0.191	+0.488	−0.208	+0.528
		85°	−0.403	+0.231	−0.482	+0.276	−0.552	+0.316	−0.616	+0.354
1 : 5	11° 19′	80°	−0.050	+0.384	−0.057	+0.440	−0.063	+0.488	−0.069	+0.528
		85°	−0.286	+0.231	−0.341	+0.276	−0.391	+0.316	−0.436	+0.354
1 : 6	9° 28′	80°	+0.020	+0.384	+0.023	+0.440	+0.026	+0.488	+0.028	+0.528
		85°	−0.204	+0.231	−0.243	+0.276	−0.279	+0.316	−0.312	+0.354

It should be noted that tension in the piles of Group 3 occurs only if, $\eta > 90° - |\alpha_3|$. The allowable load on a compression pile is P_2. Then, when the system has no rotation:

$$P_1 = n\,P_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (74a)$$

and,

$$P_3 = m\,P_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (74b)$$

If the three axes of the pile groups can be placed so that the group components of the exterior resultant are always whole multiples of P_1, P_2, or P_3, the elastic center is on the resultant. The arrangement of the piles around a group axis is immaterial. However, the number of piles in each group must equal the aforementioned whole multiple. By group axis, is understood a line parallel to the center lines of the piles in the group and going through the center of gravity of the cross-section areas of the piles. In the investigation that follows, consideration is given only to the important case in which one group of piles is vertical ($\alpha_1 = 0$).

From Table 5, in connection with general engineering considerations, possible values of m and n may be readily determined from assumed slopes, α_1, α_2, and α_3 of the three pile groups. The number of piles, a, b, and c, in each group may be found by means of a force polygon; thus, let a = number of vertical piles (slope, $\alpha_1 = 0$); b = number of battered compression piles (slope, α_2); and, c = number of battered tension piles (slope, $|\alpha_3|$).

In general, the magnitudes of the group reactions in the force polygon are not whole multiples of P_1, P_2, and P_3, but small remainders will occur in two of the forces. If such is the case, there will be a small rotating moment about

the elastic center. As it is necessary to deal with whole numbers of piles, the nearest whole multiples, a, b, and c, may be selected and new values of m, n, and η calculated.

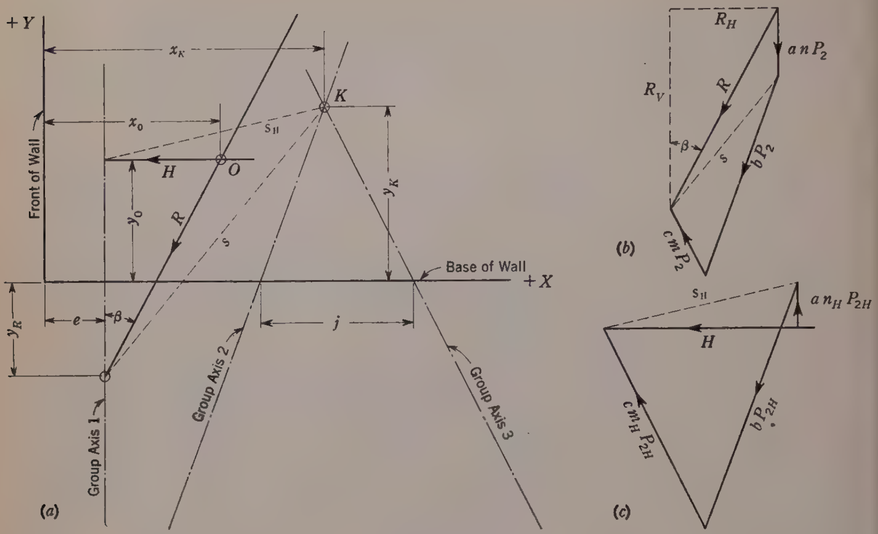

FIG. 19

The known resultant, R (Fig. 19), and the pile reactions, $b\,P_2$, $a\,n\,P_2$ and $c\,m\,P_2$, must form a closed force polygon with known directions of the sides and, at the same time, must satisfy Equations (71). Values of m and n, therefore, may be obtained in terms of the pile batters, $\alpha_1 = 0$, α_2, and α_3. The number of piles, a, b, c, and the angle of R with the vertical, β, may also be determined from this condition.

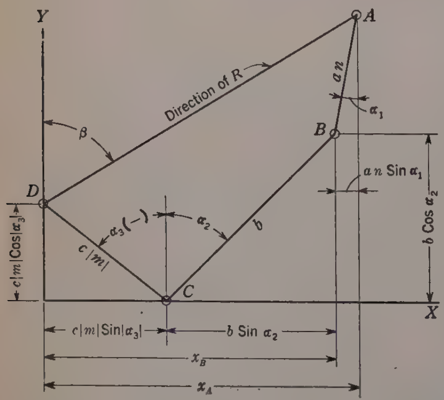

FIG. 20

Considering again the general case in which α_1 is not zero and m is negative, the force polygon will have the form indicated by $A\ B\ C\ D$ in Fig. 20. In drawing the force polygon, the scale has been chosen so that P_2=unity; hence, $A\,B = a\,n$; $B\,C = b$; and, $C\,D = c\,|m|$. (Symbol $|m|$ indicates the numerical value of m.) Fixing a co-ordinate system as indicated, the equation for Line $D\,A$ is:

$$y = x \cot \beta + c|m|\cos|\alpha_3| \quad \text{......................(75a)}$$

and the equation for Line $A\,B$ is:

$$y - b \cos \alpha_2 = \cot \alpha_1 \,[\, x - (c|m|\sin|\alpha_3| + b \sin \alpha_2) \,] \quad \text{.....(75b)}$$

At the point of intersection, A, between the two lines, Equations (75) must both be satisfied. Designating by x_A as the abscissa to Point A, and eliminating y: $x_A \cot \beta + c|m|\cos|\alpha_3| - b \cos \alpha_2 = \cot \alpha_1 [x_A - (c|m|\sin|\alpha_3| + b \sin \alpha_2)]$; or,

$$x_A = \frac{c|m|\cos|\alpha_3| - b \cos \alpha_2 + \cot \alpha_1 (c|m|\sin|\alpha_3| + b \sin \alpha_2)}{\cot \alpha_1 - \cot \beta} \quad \ldots (76)$$

If the abscissa to Point B is x_B,

$$x_A - x_B = a\, n \sin \alpha_1 \quad \ldots\ldots (77a)$$

and,

$$n = \frac{1}{a \sin \alpha_1} (x_A - x_B) \quad \ldots\ldots (77b)$$

and, since $x_B = c|m|\sin|\alpha_3| + b \sin \alpha_2$:

$$n = \frac{1}{a \sin \alpha_1} \left[\frac{c|m|\cos|\alpha_3| - b \cos \alpha_2 + \cot \alpha_1 (c|m|\sin|\alpha_3| + b \sin \alpha_2)}{\cot \alpha_1 - \cot \beta} - (c|m|\sin|\alpha_3| + b \sin \alpha_2) \right]$$

$$n = \frac{1}{a \sin \alpha_1} \left[\frac{c|m|\cos|\alpha_3| - b \cos \alpha_2 + \cot \beta (c|m|\sin|\alpha_3| + b \sin \alpha_2)}{\cot \alpha_1 - \cot \beta} \right]$$

and,

$$n = \frac{1}{a} \left[\frac{c|m|\cos|\alpha_3| - b \cos \alpha_2 + \cot \beta (c|m|\sin|\alpha_3| + b \sin \alpha_2)}{\cos \alpha_1 - \sin \alpha_1 \cot \beta} \right] \quad \ldots (78)$$

If $\alpha_1 = 0$, Equation (78) becomes,

$$n = \frac{1}{a} [c|m|\cos|\alpha_3| - b \cos \alpha_2 + \cot \beta (c|m|\sin|\alpha_3| + b \sin \alpha_2)] \quad \ldots (79)$$

Equation (79) expresses the condition that Forces $A\,B$, $B\,C$, $C\,D$, and $D\,A$, form a closed polygon. However, the deformation formulas, Equations (71), must also be satisfied. Inserting $\tan \eta$ from Equation (71a) in Equation (71b), and simplifying:

$$n = m \frac{\sin 2\,\alpha_2}{2 \cos \alpha_2 \cos|\alpha_3| \sin (|\alpha_3| + \alpha_2)} + \frac{\sin 2|\alpha_3|}{2 \cos \alpha_2 \cos|\alpha_3| \sin (|\alpha_3| + \alpha_2)} \quad \ldots (80)$$

Equation (80) shows that, with α_2 and α_3 given, and with $\alpha_1 = 0$, n is a linear function of m. If the pile foundation is to be subject to translation only and no rotation, Equations (79) and (80) must be satisfied simultaneously.

In Equation (79) the numerical value of m, designated $|m|$, has been used; and, it has been assumed that m is negative and that Pile Group 3 actually is subjected to tension. Substituting $|m|$ for m; assuming m to be negative also in Equation (80); and, equating the two expressions for n in Equations (79) and (80):

$$- |m| \frac{\sin 2\,\alpha_2}{2 \cos \alpha_2 \cos|\alpha_3| \sin (|\alpha_3| + \alpha_2)} + \frac{\sin 2|\alpha_3|}{2 \cos \alpha_2 \cos|\alpha_3| \sin (|\alpha_3| + \alpha_2)}$$
$$= |m| \frac{c}{a} \cos|\alpha_3| - \frac{b}{a} \cos \alpha_2 + |m| \frac{c}{a} \sin|\alpha_3| \cot \beta + \frac{b}{a} \sin \alpha_2 \cot \beta \quad \ldots (81)$$

After reduction:

$$|m| = \frac{\sin 2|\alpha_3| + 2\frac{b}{a}\cos\alpha_2 \cos|\alpha_3|\sin(|\alpha_3| + \alpha_2) \times [\cos\alpha_2 - \sin\alpha_2 \cot\beta]}{\sin 2\,\alpha_2 + 2\frac{c}{a}\cos\alpha_2 \cos|\alpha_3|\sin(|\alpha_3| + \alpha_2) \times [\cos|\alpha_3| + \sin|\alpha_3|\cot\beta]} \quad (82)$$

If the horizontal component of R is R_H and the vertical component, R_V:

$$\cot\beta = \frac{R_V}{R_H} \quad (83)$$

If $\beta = 90°$, so that $\cot\beta = 0$:

$$|m| = \frac{\sin 2|\alpha_3| + 2\frac{b}{a}\cos^2\alpha_2 \cos|\alpha_3|\sin(|\alpha_3| + \alpha_2)}{\sin 2\,\alpha_2 + 2\frac{c}{a}\cos^2|\alpha_3|\cos\alpha_2 \sin(|\alpha_3| + \alpha_2)} \quad (84)$$

Equations (82) and (84) hold only if m is negative, or if $\eta > 90° - |\alpha_3|$. This can be verified by computing η from Equation (71a) after substitution of $-|m|$ for m. The value of n is determined from Equation (80) after having substituted $-|m|$ for m in the same manner; thus:

$$n = \frac{1}{2}\,\frac{\sin 2|\alpha_3| - |m|\sin 2\,\alpha_2}{\cos\alpha_2 \cos|\alpha_3|\sin(|\alpha_3| + \alpha_2)} \quad (85)$$

It is evident from Fig. 20 that, for $\alpha_1 = 0$:

$$P_2\,(c|m|\sin|\alpha_3| + b\sin\alpha_2) = R_H \quad (86a)$$

or,

$$P_2 = \frac{R_H}{c|m|\sin|\alpha_3| + b\sin\alpha_2} \quad (86b)$$

Forces P_1 and P_3 are determined from Equations (74). The tangent, k_s, of the angle of Line s (Fig. 19) with the horizontal is determined by:

$$k_s = \frac{R_V - a\,n\,P_2}{R_H} = \frac{R_V - a\,P_1}{R_H} \quad (87)$$

This line intersects the line of Resultant R on Pile Group Axis 1 (Fig. 19(a)). Line s is thus determined by the location of Pile Group Axis 1 and the slope is determined from Equation (87).

The point of intersection, K, between the axis of Pile Groups 2 and 3 must fall on Line s but the actual location on s may be determined from general engineering considerations.

Pile Foundation Having Its Elastic Center at a Given Point on the Resultant.—Any pile foundation that has its elastic center on the resultant of the exterior forces will be subject to translation only. However, it may be desirable that the elastic center should be located at a particular point on R. Let the point selected be O in Fig. 19(a). Apply an arbitrary force, H, to the pier, the force being given in magnitude and direction and passing through Point O. This force preferably should be horizontal. Compute $|m_H|$ from Equation (84) and n_H from Equation (85). To determine the slope of Line s_H (corresponding to

Line s) rewrite Equation (87) for $R_V = 0$ and $R_H = H$; thus:

$$k_H = -\frac{a\,n\,P_2}{H} \qquad (88a)$$

Inserting P_2 from Equation (86b):

$$k_H = -\frac{n_H\,a}{c\,|m_H|\sin|\alpha_3| + b\sin\alpha_2} \qquad (88b)$$

Draw Line s_H with a slope, k_H, through the point of intersection of H and Pile Group Axis 1. The intersection of s_H and s is the particular point, K, at which Axes 2 and 3 must intersect to insure an elastic center at the particular point, O.

It can be shown in general that the lines, s, corresponding to the resultants, R (all of which go through the same point) also intersect in a single point. The points, O and K, are uniquely interdetermined. Instead of choosing a horizontal force through O for the determination of Point K, a force may be chosen which corresponds to some alternate possible loading on the structure. In general, it is advantageous to choose Point O at or near the points of intersection of the resultants of the various loadings so as to reduce the magnitude of the rotating moment.

Pile Foundations Having a Given Arrangement of the Battered Piles.—It is often of importance to determine the pile foundation so that the distance, j, between the points of intersection of Axes 2 and 3 with the base of the pier has a given value (Fig. 19(a)). If x_K and y_K denote the abscissa and ordinate to Point K, x_O and y_O, the corresponding quantities for Point O, are:

$$j = y_K\,(\tan\alpha_2 + \tan|\alpha_3|) \qquad (89)$$

or:

$$y_K = \frac{j}{\tan\alpha_2 + \tan|\alpha_3|} = j\,\frac{\cot\alpha_2\cot|\alpha_3|}{\cot\alpha_2 + \cot|\alpha_3|} \qquad (90a)$$

$$x_K = \frac{y_K - y_R}{k_s} + e \qquad (90b)$$

and,

$$y_O = y_K - k_H\,(x_K - e) \qquad (90c)$$

Distance e, the abscissa to the point of intersection of Axis 1 and the base of the pier, is usually fixed by engineering considerations; and y_R denotes the ordinate to the point of intersection of R with Axis 1. For the condition indicated in Fig. 19(a), y_R is negative and should be so inserted in Equation (90b); and k_s is determined from Equation (87) and k_H from Equation (88b). Since the elastic center is located on R, Quantity y_O determines that point uniquely; and x_K and y_K determine the point of intersection of Axes 2 and 3.

By fixing the elastic center, O, at some particular point of the resultant, R, or by fixing, beforehand, the characteristic quantity, j, of the pile foundation, and from this quantity computing the position of O on R, a satisfactory pile foundation can be designed to resist any given load. The method may be used also in case it is desired to design a pile foundation so that one or two different resultants produce a definite rotating moment, M, about the elastic center. In the latter case the resultant is translated a distance, $\frac{M}{R}$. The

additional pile loads due to the rotating moment are as given by Equation (37) of the paper.

Design Procedure.—The following procedure has particular reference to the design of a retaining wall for which the horizontal component of the earth pressure is independent of the width of the base. Assume that the elevation of the pile tops, the length of the unit of the wall for which the pile foundation is to be designed, the permissible pile loads, the horizontal component of the exterior forces, and the length and cross-section area of the piles, are given.

It is required to determine the width of the base slab, the resultant vertical component of the exterior forces, the batter of the piles, the number of piles, the pile loads, and the arrangement of the piles. The steps in this procedure are:

(1) Determine the horizontal component, R_H, of the exterior forces.

(2) Select the pile batters.

(3) By varying the width of the base slab, select a value of the vertical component of the exterior forces, R_V, so that the corresponding R has a direction approximately equal to the direction of the battered compression piles.

(4) Determine the required number of piles by drawing the force polygon demonstrated in Fig. 19(*b*). In determining the required number of piles in Groups 1 and 3, approximate values of $|m|$ and n should be taken from Table 5.

(5) Compute the exact value of $|m|$ from Equation (84) checking to be certain that $\eta > 90° - |\alpha_3|$.

(6) Compute the exact value of n from Equation (85).

(7) Compute Unit Pile Load P_2 of a battered compression pile from Equation (86*b*).

(8) Draw Line s by computing its slope from Equation (87).

(9) Apply a horizontal load, H, to the pier, to be placed so that it intersects R at the desired location of the elastic center. Evaluate $|m_H|$ and n_H from Equations (84) and (85), respectively.

(10) Draw Line s_H by computing its slope from Equation (88*b*).

(11) If it is desired to fix the elastic center at a given point, determine Point K as the point of intersection of s and s_H.

(12) If it is desired that the distance, j (Fig. 19(*a*)), should have a given magnitude, determine the values of y_K, x_K, and y_O from Equations (90), respectively, or by means of graphical construction.

(13) Arrange the piles around their respective axes according to general engineering considerations.

Conclusions.—The author has presented a method for direct determination of the number of piles required in a pile foundation for a pier subject to two loading conditions, provided the foundation has piles in two directions only.

In the discussion presented herein, the writer has extended the method to cover pile foundations having piles in three directions. Complete formulas and method of procedure have been given for the case in which one of the three pile groups consists of vertical piles. This special case is of considerable practical importance in connection with the design of quay walls and other retaining walls.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

MOTOR TRANSPORTATION—A FORWARD VIEW
A SYMPOSIUM

Discussion

BY MESSRS. ROBERT B. BROOKS, AND H. GEORGE ALTVATER

ROBERT B. BROOKS,[35] M. AM. SOC. C. E. (by letter).[35a]—In the final analysis, the place of the express highway in a modern road system must be determined by its economic justification.

The most important economic phase of the paper by Mr. MacDonald is that in relation to the continued construction of super-highways. His idea is that present policies indicate that the location of super-highways will be integrated with population centers and that the layout will not be on the transcontinental basis.

It is undoubtedly true that in Germany and Italy the express highways are built for protection and aggression as well as for peace-time traffic. In the United States express highways are built to satisfy the demand of peace-time traffic, as well as to provide highways that can be used for handling men and equipment in time of war. This protection did not exist twenty years ago, as was shown definitely, time after time, by the wrecking of highways by even limited troop movements during the World War. There can be no doubt that there is a definite place for the express highway in a modern road system.

In January, 1932, the United States had a total of 3 790 miles of extra-wide highways in its State systems and not included in the mileage of city streets and parkways.[36] By extra-wide highways are meant those having more than two lanes. There were 2 230 miles of three-lane width, 1 385 miles of four-lane width, and 175 miles of five-lane width and greater.

Five years later, and bringing this date up to July 1, 1937, there is a total of 7 999 miles of important highways exceeding two-lane width, of which

NOTE.—This Symposium was presented at the meeting of the Highway Division, Detroit, Mich., July 21, 1937, and published in June, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1938, by Messrs. F. Lavis, Edgar Dow Gilman, George Hartley, Robert Kingery, R. L. Morrison, and Roy F. Bessey; and October, 1938, by Bruce D. Greenshields, Assoc. M. Am. Soc. C. E.

[35] Cons. Engr.; Member, Missouri State Highway Comm., St. Louis, Mo.

[35a] Received by the Secretary October 19, 1938.

[36] *Public Roads*, November, 1932.

4 703 are three-lane, 3 096 miles are four lane, and 201 miles are five and six-lane.[37] This increase in width is significant of the trend in highway construction. Within that same time the genuine planning of modern highway systems has been given exhaustive study, under the basic assumption that they should be planned to do the greatest good for the greatest number.

State traffic surveys should reveal not only the traffic volumes, and other physical information, but should include a description of financial status and explanations of how the citizens are to pay for highways in the future. Consider Missouri, for example. The total mileage of all the roads in the State is 104 850. The State highway system, as now designated, contains a total of 17 193 miles. Of this total there are 1 651 miles in the primary system, 5 877 miles in the secondary system, and the remainder in the supplementary system with its traffic relief, park connections, etc. In other words, the State highway system in Missouri specifically consists of 16.4% of the total road mileage in the State which, however, does carry more than three-fourths of all the traffic when measured in vehicle-miles per year. Carrying this computation a step further, the several systems of highways in Missouri, in the survey report of 1934, showed the following percentages of traffic:

System	Highway mileage	Vehicle mileage
United States highways	3.6	48.1
State highways	4.6	18.5
Supplementary	4.1	8.0
County roads	87.7	25.4
Total	100.0	100.0

Although proper consideration must be given to the farm-to-market roads and to the secondary road system, it is readily apparent that the main highways carry by far the greater volumes of traffic. This is as true in other States as it is in Missouri.

The era of toll roads in the United States is passed. Although individual experiences may warrant the impression that the user would be willing to pay an extra sum for using express highways between centers of population, the cold facts are that there are not enough motorists to amortize the cost of building an express highway in the United States. This is proved by the fact that most motorists will drive many miles out of their way to avoid paying even a small toll on a bridge which might otherwise save them considerable time as well as the price of the bridge fare.

If the cost of the express highway is to come from gasoline tax and vehicle license fees, with added grants from the Federal Government, engineers must "cut the coat to fit the cloth," and in so doing build express highways in such a way that they will handle those traffic volumes in, around, and near metropolitan areas where the expense of construction can be justified, not only from an engineering standpoint but from that of public opinion.

With the proceeds of most of the State bond issues for road purposes already spent, road programs seem to be on a "pay-as-you-go" basis. This

[37] *American Highways*, July, 1937.

means a further economic justification rather than a lavish expenditure to take care of anticipated needs.

Highways are often called traffic arteries through which streams of automobiles flow. The comparison, of course, is to that most perfect system of veins and arteries which takes care of the circulation of the blood. Comparing traffic flow to the distribution of water, the largest pipes are next to the source of supply, or reservoir. As the water is carried to various sections, the sizes of the pipes are lessened in deference to the demand for water by the consumers. The system of arteries and veins in the human body shows very definitely that the need for the larger arteries and the larger veins is near the heart, and the size of each is smaller and smaller as it gets away from those parts of the body that need circulation least.

It is self-evident that the pressure of traffic is from the centers of congestion in metropolitan areas outwardly, just as water is distributed from a reservoir, and just as blood is circulated in the body of an individual.

"Flow" maps invariably show that traffic demands wider traffic-ways near centers of population. The same maps also show that the two-lane highways are sufficient to take care of traffic needs a reasonable number of miles away.

Although the out-of-State farmer can see no reason why his gas tax money should go for the building of an express highway from Chicago, Ill., to another locality, neither does the urban dweller see any reason why his tax money and that of his neighbors in the large city, which may pay three-fourths of the entire cost of constructing highways in the State, must be used in building farm-to-market roads which he, the urban dweller, may never use.

The opinion has been expressed that large cities must construct express highways in order to exist. The writer believes that various State traffic surveys will supply the factual data which will educate every one, whether urban or rural dwellers, as to the mutuality of traffic needs so that gas taxes and vehicle license fees can be spent on a network of traffic arteries, which will take all classes of roads into consideration.

During the eight years, 1925–1933, a definite system of streets was laid out in the City of St. Louis, Mo. These streets were divided roughly into three classes: 60-ft streets, 80-ft streets, and 100-ft streets. A progressive system of construction each year saw various sections completed, so that to-day with the street plan practically completed, traffic flow maps show, definitely, the value of the various widths of streets. Consider the planned design of what is known as US 40TR, in Missouri, which extends from St. Louis westward about 40 miles to Wentzville, Mo. It is a two-lane highway from Wentzville to the Missouri River; a three-lane highway from that point to Chesterfield; a four-lane contiguous highway from Chesterfield to Bellefontaine; and then 2 two-lane highways divided by a 30-ft parkway to a point close to the City of St. Louis. At this point it is necessary to converge into a subway running three miles inside the city limits to Vandeventer Avenue, where traffic is then poured into two or three streams for distribution.

Through bitter experience highway engineers in Missouri have already learned how costly it is to obtain right of way sufficiently large for an express highway. The longer the acquisition of land is delayed, the more it will cost.

Therefore, on express highways they have, and are, securing a 200-ft right of way which will take care of future traffic needs, although the present-day construction of extra-wide highways may not be warranted at the present time.

Although many engineers would like to see the general acceptance of four-lane highways instead of two lanes, it seems self-evident that there is not economic justification for such an expenditure in many cases, and, although there may be some accidents caused by a third lane, the three-lane traffic is a transition from a two-lane highway to the express highway. Any one who doubts this point should note the immediate relief afforded when driving in holiday traffic on a two-lane highway almost at the moment it passes into a three-lane highway.

State traffic surveys now being completed and analyzed will show where express highways should be located and, although there may not be economic justification for the construction of an express highway now, it certainly is good planning to acquire land for the subsequent building of these roads when, if, and as, the need arises.

At its meeting in July, 1937, the members of the Missouri State Highway Commission were sufficiently impressed with this idea on U. S. Route 54, between Jefferson City, Mo., and the Lake of the Ozarks, that they instructed the Engineering Department to arrange for the acquisition of sufficient right of way to provide for the building of an express highway of extra width sometime in the future. At present, only a two-lane highway, leading south from Jefferson City for a few miles, will be constructed. Missouri needs express highways to take care of the outward pressure of traffic from its large cities. The writer cannot agree that there is a need for express highways up and down the land where the traffic itself cannot show an economic justification. He does agree most emphatically that engineers must plan wisely to secure sufficient right of way in order to add more lanes to take care of future traffic demands.

Mr. R. E. Toms, Chief, Division of Design, U. S. Bureau of Public Roads, recently presented data to demonstrate that from 95% to 97% of the State highway mileage in the United States may never progress in improvement beyond a two-lane highway.[38] If it is desirable to improve such a small percentage of the highway system as express highways, then let it be realized from the standpoint of economic justification, that the function of an express highway in a modern road system is to take care of the outward pressure of traffic from metropolitan areas to the extent that this extra pressure exists, and not as a continuous super-highway connecting centers of population.

If any further evidence is needed to show that the location of super-highways should be integrated with respect to population centers and that their layout should not be on a transcontinental basis, it is only necessary to observe the restricted use of the State and Federal highways immediately outside the orbit of metropolitan areas as ably shown by Mr. MacDonald.

H. George Altvater,[39] Assoc. M. Am. Soc. C. E. (by letter).[39a]—Referring to "Trunk-Line Highways in Metropolitan Areas," Mr. Smith states

[38] *American Highways*, January, 1937, p. 12.

[39] Cons. Engr., Denver, Colo.

[39a] Received by the Secretary November 19, 1938.

that highways must serve the area as trunk lines for public utilities and that inadequacy of right of way will affect the utilities. The practise of laying pipes beneath the traveled roadways of streets was inaugurated in the days of horse-drawn wagons on dirt roads, and then it caused little inconvenience in digging up the streets. With the carefully designed pavement of to-day, however, there is no reason why each block of it must be repeatedly trenched through and patched numerous times during its life.

At the time of the World War, the United States found itself so thoroughly unprepared to supply munitions that the only experienced engineering organization in the nation that was qualified to build explosives plants could undertake to build only about one-half the necessary capacity, and the remainder was built by entirely new inexperienced organizations. Under these circumstances, the matter of utilities in streets was a serious cause of delay. The main plant transportation, at Nitro, near Charleston, W. Va., where one of the two American smokeless powder plants was constructed, consisted of 45 miles of narrow-gage railroad, which was responsible for the results obtained. In the deep mud the tractors and even horses became bogged down, so that oxen were resorted to for moving heavy machinery. It was not possible to build roads first because the streets were torn up for utilities.

There were untreated industrial water lines, treated domestic water lines, high pressure fire lines, steam lines, industrial sewer lines, domestic sewer lines, storm sewer lines, gas lines, electric power lines, telephone and telegraph lines, sulfuric acid lines, nitric acid lines, caustic soda lines, ether lines, and alcohol lines. Their construction in the streets obstructed the delivery of essential machinery. Time was the only consideration, because in July, 1918, the Allied Armies were using explosives about six times as fast as they could be produced by all plants. The separation of rights of way for utilities would have permitted road construction to be done previously or concurrently. The writer recommends to industrial city planning engineers that an attempt be made to provide separate rights of way for utilities on new developments.

This is in no way a reflection upon the excellent work of those who were responsible for the miraculous speed in the construction of the plant at Nitro, because they produced their $75 000 000 job in about the same time as the slightly larger Nashville (Tenn.) job was done by the explosives specialists.

Although the presence of utilities in streets contributes greatly to the engineering difficulty and cost of subways in the large cities there does not seem to be any feasible way to avoid it; but in new developments where land is relatively cheaper a separation of the utilities in the rights of way should be possible.

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DISCUSSIONS

LATERAL EARTH AND CONCRETE PRESSURES

Discussion

By Messrs. Jacob Feld, M. G. Spangler, and Raymond D. Mindlin

Jacob Feld,[21] M. Am. Soc. C. E. (by letter).[21a]—A careful analysis of this paper fails to disclose very much new material or reasonably reliable methods for the determination of lateral earth or concrete pressures. It is sincerely hoped that any attempted use of the formulas or recommendations announced by the authors will be made only after a careful study of the limitations on such results. These limitations are stated in the paper, but not in such manner as to warn, sufficiently, the reader who may attempt to apply the results to future design problems.

In the "Introduction," three "phases" of the analysis of lateral pressures are enumerated, the first two dealing with lateral pressures of unfractured and fractured banks, the third being a study of the effect of surface loading. The third case should be discussed as an additional loading to each of the first two and also the possibility that such addition to the first case, unfractured bank without surcharge, will bring it into the second class. In general, if a wall, whether part of a retaining wall, bin, sheathing, or concrete form, has no elastic, plastic, or displacement motion, no pressure can be measured. If the motion is purely an elastic strain, the corresponding elastic strain in the earth can be measured by the elastic theory if the moduli of elasticity of wall and earth are fairly similar.

In the discussion of Phase I, the assumption that there are no fractured surfaces (both internal and external) is equivalent to an assumption that the internal stresses in the earth are of such nature that at no point is the tensile strength greater than the elastic limit in tension. The total distorsion, elastic and displacement, must not exceed the strain at the point of elastic limit in tension. In an elastic make-up of elastic members, the less rigid will take

Note.—The paper by Lazarus White and George Paaswell, Members, Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1938, by A. E. Cummings, M. Am. Soc. C. E.; and December, 1938, by Messrs. G. Hennes, Robert F. Legget, and Charles Terzaghi.

[21] Cons. Engr., New York, N. Y.

[21a] Received by the Secretary November 14, 1938.

less than its share of the load, as determined from static computation. However, as stated in the paper, the experimental work of Professor Terzaghi indicates that extremely minute displacement, far less than the smallest deformation of any economically designed sheathing or retaining wall, will result in a reduction of the measured pressure to amounts indicated by the classic earth-pressure theories; and this case is Phase II, as stated by the authors in the "Introduction."

The typical section of a fracture surface, Fig. 1, has been found by several previous writers on the subject, notably E. G. Haines[22] and H. G. Moulton,[23] Members, Am. Soc. C. E. This type of fracture is usual in slides.

In suggesting the assumption of different characteristics for soil pressure computations for members in the same sheeting and bracing unit, the authors should explain that the requirements of statics must not be forgotten. The total load on the wales can only come from the sheeting, and the total load on the braces or ties can only come from the wales. However, in computing reactions, the effect of continuity as well as the deformation of supports should be considered. The total load in all the braces or ties cannot be more or less than the total load on the sheeting.

In the discussion of the "Theory of Design—Sheeting and Bracing," it should be emphasized that Equation (1) applies only when the elastic modulus for tension as well as compression is not exceeded, and the strain and stress relationship is linear. In the design of retaining walls, the magnitude of the lateral pressure is relatively less important than its direction and the location of the point of application; these last two items are not considered in the paper.

The typical distribution of lateral pressures against a vertical face due to a surface load as shown in Fig. 2 is a fiction. No such distribution has ever been found; nor can any material be imagined in which the maximum effect is found at the top of the wall. The discontinuity of the earth surface must be considered even in the case where elastic equations may be used. Experimental results on this problem have been published by M. G. Spangler,[24] Assoc. M. Am. Soc. C. E.

A complete paper on the use and limitations of elastic theories on the determination of earth pressures was published by the late J. H. Griffith,[25] M. Am. Soc. C. E.

The limitations to the use of Equations (23) to (27),[26] evaluating lateral pressure due to various types of loading, are given as follows: "The lateral pressures, as given by Equations (23) to (27), are assumed to be distributed along rigid supports yielding no more than the elastic material which it supports." If any case of retaining wall, bin, or sheeting, can be found to comply with that limitation, the equations and tables are of value.

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[22] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1513; also, *Engineering News-Record*, Vol. 86 (1921), p. 89.

[23] *Loc. cit.*, Vol. LXXXIII (1919–1920), p. 299; also, Am. Inst. Min. and Met. Engrs. (1920), Section 13, Paper No. 158.

[24] "Horizontal Pressures on Retaining Walls Due to Concentrated Surface Loads," by M. G. Spangler, *Bulletin No. 140*, Iowa Eng. Experiment Station, 1938.

[25] "Dynamics of Earth and Other Macroscopic Matter," by J. H. Griffith, *Bulletin No. 117*, Iowa Eng. Experiment Station.

[26] *Proceedings*, Am. Soc. C. E., May, 1938, p. 859.

The theory developed for concrete pressures is not in agreement with the results of the experimental works by F. R. Shunk[27] and by E. B. Smith.[28] As the authors state (see heading, "Concrete Pressures"), pressures were obtained by translating extensometer readings based on an assumed distribution of pressure from sheeting panel to stud and wales and, in turn, to tie-rod. If the effect of continuity of studs and wales on reactions is not considered, errors in the assumed pressures will result.

For those who may be interested in a further study of the elastic theories as applied to earth pressures, reference should be made to the reports of the Special Committee of the Society to Codify Present Practice on the Bearing Value of Soils for Foundations.[29]

With the limitations included in the requirement of elasticity taken into consideration, the authors' paper is a general solution of problem. The principal objection to its applicability is the non-existence of earth materials which may be classified as elastic.

M. G. Spangler,[30] Assoc. M. Am. Soc. C. E. (by letter).[30a]—Although retaining walls and allied types of construction are among the oldest and simplest kinds of structures with which engineers are concerned, there are many phenomena, related to the overturning and translating forces which retaining walls must be built to withstand, that are only dimly understood. It is to the credit of the Engineering Profession that its members are continually seeking further knowledge in regard to these phenomena, as is attested by the very extensive bibliography of this subject which is to be found in the literature. This paper is a valuable contribution to retaining wall literature and is especially noteworthy in that the authors have discarded the classical wedge theories and have based their argument upon the fundamental laws of the theory of elasticity, an approach with which the writer is wholly in sympathy.

The authors very properly divide their subject into three fundamental phases: (I) Lateral pressures when the wall is rigid enough to prevent fracture of the back-fill mass; (II) lateral pressures when the wall yields enough to permit fracture planes within the back-fill; and (III) lateral pressures induced upon the wall by surface loading. This discussion will be confined mainly to the last of these phases.

In 1936, rather extensive experiments were reported by the writer[31] in which the lateral pressures on retaining walls caused by the application of concentrated loads on the surface of the back-fill were measured. Typical results of these experiments are shown in Fig. 9. Fig. 9(*a*) shows the results of some similar

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[27] "Pressure of Concrete in Forms," by F. R. Shunk, *Professional Memoirs*, Corps of Engrs., U. S. Army, Vol. 1, 1909, pp. 247–260.

[28] "Pressure of Concrete in Forms," by E. B. Smith, *Proceedings*, Am. Concrete Inst., February 15, 1920.

[29] *Proceedings*, Am. Soc. C. E., 1915, 1916, 1917, 1920, 1921, 1922, 1923, and 1925.

[30] Research Associate Prof., Iowa Eng. Experiment Station, Iowa State Coll., Ames, Iowa.

[30a] Received by the Secretary November 15, 1938.

[31] "Horizontal Pressures on Retaining Walls Due to Concentrated Surface Loads," by M. G. Spangler, *Bulletin No. 140*, Iowa Eng. Experiment Station, Iowa State Coll., Ames, Iowa, April, 1938; also, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Graduate School of Eng., Harvard Univ., Cambridge, Mass., Vol. 1, 1936, pp. 200–207.

experiments[32] performed at Erdgenössichen Technischen Hochschule at Zurich, Switzerland, in 1929 which are in substantial agreement with those conducted at Ames, Iowa.

This experimental evidence confirms the applicability of the Boussinesq type of equation to the qualitative distribution of pressure on a retaining wall due to a concentrated load (such as a truck wheel) acting on the surface of the back-fill. Quantitatively, however, the experimental pressures were much

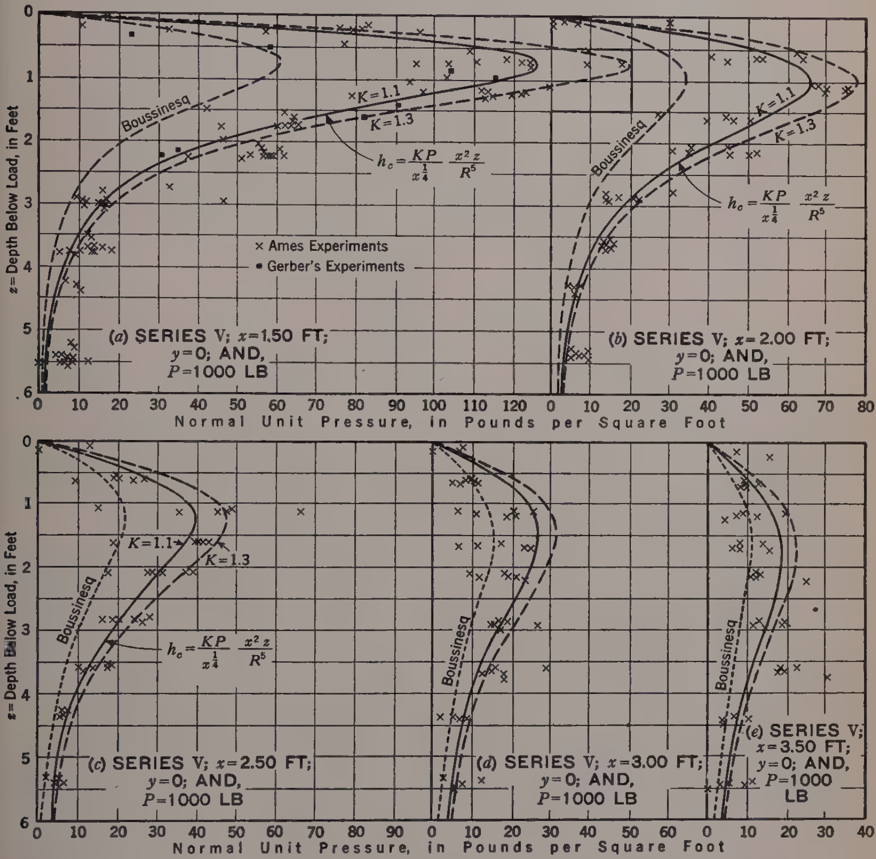

FIG. 9

greater than those indicated by the Boussinesq theory when the strain-interrupting effect of an interposed retaining wall is ignored, and the authors' assumption that this effect can be neglected without serious error is not confirmed. The average pressures measured in the experiments were about 2.1 times as large as the Boussinesq pressures calculated with Poisson's ratio = 0.5 when a wheel load was placed 1.5 ft from the wall and this factor ranged to about 1.7 with the load 3.0 ft from the wall. These greater pressures resulted from the fact

[32] "Untersuchungen über die Druckverteilung im örtlich belasteten Sand," by Emil Gerber, Diss. a-g., gebr., Leemann, Zurich, 1929.

that the relatively rigid retaining wall suddenly interrupted the lateral strains in the gravel back-fill mass at the plane of the back face of the wall and caused an accumulation of stress over and above that which would have existed at this same plane if the wall had not been present and the gravel mass had been indefinite in extent. Furthermore, this accumulation of stress was greater when the load was nearer the wall, because the lateral strains at this plane were greater for this load position, and their interruption caused a relatively greater effect.

The experiments also show that the position of the point of maximum pressure on a wall is a function of the distance from the wall to the load and that it is independent of the height of the wall. The maximum pressure occurs at a point whose distance below the plane at which the load is applied is about one-half the distance from the wall to the load, rather than at the top of the wall as shown in Fig. 2 of the paper. This phenomenon is in accord with the Boussinesq theory, as discussed by D. P. Krynine,[33] M. Am. Soc. C. E.

An empirical formula of the Boussinesq type which seems to fit the data of these experiments is:

$$p_w = \frac{\kappa\, P\, x^2\, z}{x^n\, \rho^5} \qquad \text{(40)}$$

in which p_w = normal unit pressure on the wall at any point; P = applied wheel load; x = distance from load to back face of wall; y = lateral distance from any point on the wall to the normal vertical plane containing the load; z = vertical distance from any point on the wall to the horizontal plane containing the load; ρ = radius vector = $\sqrt{x^2 + y^2 + z^2}$; and, κ and n = empirical constants = 1.1 and 0.25, respectively, in these experiments.

The co-ordinate system for Equation (40) is the same as given in Fig. 3(*a*) of the paper. By considering a uniformly distributed line load parallel to the wall to be a series of closely spaced equal concentrated loads, this formula may be integrated in the Y-direction to yield:

$$p_l = \frac{2\,\kappa\, P\, x^{(2-n)}\, z}{\rho_1^4}\left[\frac{\rho_1^2\, y_0}{3\,(\rho_1^2 + y_0^2)^{1.5}} + \frac{2\, y_0}{3\sqrt{\rho_1^2 + y_0^2}}\right] \qquad \text{(41)}$$

in which: y_0 = one-half the length of the line load; and $\rho_1 = \sqrt{x^2 + z^2}$. When y_0 is equal to zero,

$$p_l = \frac{1.33\,\kappa\, P\, x^{2-n}\, z}{\rho_1^4} \qquad \text{(42)}$$

In one of the experiments, a uniformly distributed line load was applied 2 ft back of, and parallel to, the retaining wall. The lateral pressures measured on the back face of the wall at a vertical element opposite the mid-point of the load are shown in Fig. 10, together with the pressures indicated by Equations (41) and (42), and by the Boussinesq theory. The close agreement between the measured values of pressure and those calculated by the integrated empirical

[33] Discussion of Paper No. *J-1*, by D. P. Krynine ("The Distribution of Normal Pressure on a Retaining Wall Due to a Concentrated Surface Load," by M. G. Spangler, Vol. 1, pp. 200–207), *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Graduate School of Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. 3, pp. 159–160.

equation proves the validity of the principle of superposition and the premise that a line load acts as a series of closely spaced concentrated loads.

No experiments have been conducted as yet with area loads applied at the back-fill surface. In the meantime, with the line load experience as a background, it seems tenable to treat an area load as a series of parallel line loads. However, the integration of Equation (41) or (42) in the X-direction is a difficult process and has not yet been accomplished; but, by dividing an area load into a series of parallel strip loads about 1 ft wide, the pressure on a wall can be determined by arithmetical summation without an unreasonable amount of labor. It is the writer's opinion that strip elements 1 ft wide will be small enough to yield results of sufficient accuracy when dealing with stress trans-

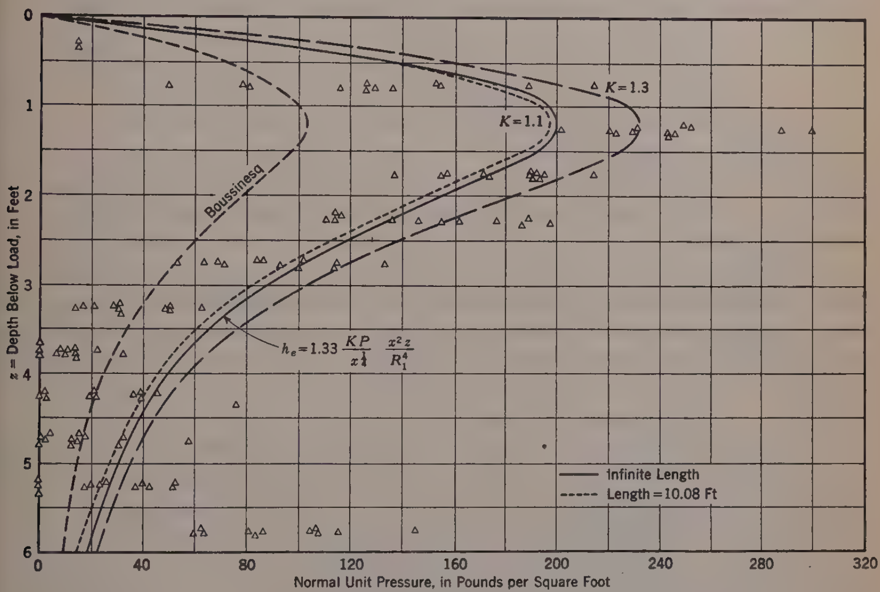

FIG. 10.—SERIES III; LINE LOAD PARALLEL TO WALL, $x = 2.00$ FEET AND $P = 1\,000$ POUNDS PER LINEAR FOOT

mission through materials having the degree of heterogeneity which most retaining wall back-fills possess.

Furthermore, the fact of heterogeneity of back-fill materials raises the question of the feasibility of introducing Poisson's ratio into any lateral pressure formulas, whether empirical or rational, because of the difficulty of determining suitable values of this constant. An impracticable number of field determinations of the ratio would be required for a specific back-fill to obtain a suitable average value for use in formulas. Rather, the effect of Poisson's ratio should be included in the empirical disposable κ in Equations (40), (41), and (42), and working values of this factor should be determined by extensive experiments on various kinds of back-fills and retaining walls. The experiments mentioned herein are fairly adequate for the one kind of back-fill and a light wall, but they need to be duplicated for many other combinations of back-fills and walls.

Finally, the writer wishes to close this discussion by suggesting that a new approach to the problem of retaining wall pressures due to earth back-fill might be made in the light of the remarkable qualitative resemblance between the measured pressures due to surface loads and those calculated by the classical Boussinesq equations for stress distribution in an elastic solid. In such an hypothesis, the weight of each small increment of back-fill material might be considered to be a concentrated load which would transmit a horizontal pressure to a retaining wall through the mass of fill material lying below the increment. The sum of the horizontal pressures due to all such increments of volume above any point on a retaining wall would be the pressure on the wall at that point.

The general form of a mathematical expression of this idea would be:

$$p_z = \kappa\, w \int_0^z \int_{-\infty}^{+\infty} \int_0^{\infty} \frac{z_i\, x^{(2-n)}}{(x^2 + y^2 + z_i^2)^{2\cdot 5}}\, dx\, dy\, dz_i \quad \ldots\ldots\ldots (43)$$

in which: p_z = horizontal pressure on a retaining wall at any depth, z, below the surface, due to back-fill material; w = unit weight of back-fill material; and z_i = vertical distance from any incremental volume of back-fill down to a depth, z.

For restricted volumes of back-fill material other appropriate limits than those shown in Equation (43) should be used. A consideration of this proposition indicates that the pressure on the wall may be affected by the distance that the back-fill extends back of the wall to a much greater extent than is indicated by the orthodox wedge theories.

RAYMOND D. MINDLIN,[34] JUN. AM. SOC. C. E. (by letter).[34a]—In applying the mathematical theory of elasticity to the study of a complex stress distribution, there are certain fundamental rules which must be observed. For the purposes of this discussion, it will suffice to consider only three of these, known technically as the stress equations of equilibrium, the strain compatibility conditions, and the boundary conditions.

The equilibrium equations represent a set of laws which control the variation of stress from point to point in the solid in such a manner as to assure the existence of a state of equilibrium between each element of the solid and the surrounding elements.

Proper observance of the compatibility conditions assures geometric continuity of the deformed solid; that is, there will be no overlapping or separation of adjacent elements if these conditions are fulfilled.

The foregoing requirements are satisfied by infinite numbers of stress distributions. Which of these distributions will actually exist in the solid depends upon the shape of the body and the forces acting on it. The "boundary conditions" constitute a formulation of the latter conditions.

A simple example will illustrate these principles. Consider a beam that is subjected, successively, to the following actions: (1) Simply supported at its

[34] Instr., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.
[34a] Received by the Secretary November 23, 1938.

ends and carrying a concentrated load at its center; (2) fixed at its ends and carrying a load uniformly distributed over its length; and (3) constrained along a certain part so that it will remain rigidly undeformed while loads are applied to other parts. It is clear that each of these cases will result in a different stress distribution but all of them may satisfy the equilibrium and compatibility conditions. No designer could properly maintain that the stresses calculated for one case would apply to the others.

An analogous situation to the foregoing presents itself in the paper. The authors have taken several well-known solutions of the elasticity equations, valid for the particular boundaries and boundary conditions for which they were intended, and have applied them to very different boundaries and boundary conditions. The stresses, of course, still satisfy the equilibrium and compatibility conditions but, without more supporting evidence than is given in the paper, they can no more be said to describe the states of stress that exist in the problems presented than do the stresses in a uniformly loaded, fixed-end beam coincide with the stresses in a simply supported beam carrying a concentrated load. In fact, certain key solutions are noted subsequently which indicate that the conclusions of the authors may involve under-estimates as great as 100%, and almost unlimited over-estimates.

Gravitational Loading.—Turning to the equations offered in the paper, it is observed that the authors state that "for an elastic solid subject to its own weight alone, the total lateral pressure at any depth, y, below the surface is given by [Equation (1)]." In the light of the foregoing discussion, it is necessary to investigate the boundary conditions to which such a pressure might conform.

First, a body acted upon by its own weight alone would move with gravitational acceleration and would be entirely unstressed. If the body is to remain stationary, it must be supported. One method of support is by means of forces applied to the surface of the body. This implies specification of the shape of the solid—that is, a description of its boundaries. Furthermore, it implies specification of the conditions of stress or displacement on these boundaries. A sphere resting on a horizontal plane support will not have the same stress distribution as a cube held in a similar manner; and a sphere resting on three point supports will have still a different stress distribution.

In writing Equation (1), the authors perhaps had in mind the semi-infinite solid. In order to study the validity of the equation in this case, it is necessary first to describe, completely, the state of stress of which Equation (1) is only a part.

The semi-infinite solid is one having as its upper boundary a horizontal plane of indefinite extent, the solid itself extending indefinitely downward from the plane. Consider a system of rectangular co-ordinates, x, y, z, oriented so that the $X-Y$-plane coincides with the plane boundary, and the Z-axis penetrates vertically downward into the solid. Let σ_x, σ_y, σ_z, τ_{yz}, τ_{zx}, τ_{xy} represent the six components of stress at any point in the solid, in accordance with an accepted notation.[35] Then, the complete specification of stress,

[35] "Theory of Elasticity," by S. Timoshenko, N. Y., 1934.

partially represented by Equation (1), is:

$$\sigma_x = - w\, z \frac{\mu}{1 - \mu} \quad \text{(44a)}$$

$$\sigma_y = - w\, z \frac{\mu}{1 - \mu} \quad \text{(44b)}$$

$$\sigma_z = - w\, z \quad \text{(44c)}$$

and,

$$\tau_{yz} = \tau_{zx} = \tau_{xy} = 0 \quad \text{(44d)}$$

The strains, ϵ_x, ϵ_y, ϵ_z, γ_{yz}, γ_{zx}, γ_{xy}, corresponding with the six components of stress, are found by using Hooke's law for an isotropic material:

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y + \sigma_z)\right] \quad \text{(45a)}$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \mu(\sigma_z + \sigma_x)\right] \quad \text{(45b)}$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_x + \sigma_y)\right] \quad \text{(45c)}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \quad \text{(45d)}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} \quad \text{(45e)}$$

and,

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \quad \text{(45f)}$$

Substituting in Equations (45) the values of the stress components given in Equations (44), it is found that,

$$\epsilon_x = \epsilon_y = 0 \quad \text{(46)}$$

Hence the lateral pressure given by Equations (44) (and Equation (1)) results from the assumption that there is no lateral displacement or deformation of any vertical plane in the solid. In a semi-infinite solid this may be interpreted as setting up a boundary condition. Since many other types of lateral restraint may be postulated, it is apparent that Equation (1) cannot be accepted as general even for the semi-infinite solid.

The state of stress in a large earth mass, approximating the specifications of a semi-infinite solid, and subjected to gravitational loading only, cannot be determined definitely from the theory of elasticity alone. The stress situation that exists in a particular earth mass depends upon the past history of the body. Experiments, interpreted by means of elasticity theory, might give some indications, but not theory alone.

Poisson's Ratio and Hooke's Law.—Following Equation (1) the authors make certain statements concerning Poisson's ratio which are not quite accurate. They state, "if it were greater than 0.5, the material would not resist distortion; and if it were less than − 1, it would not resist compression."

These limits are determined from a consideration of the principle of conservation of energy.

Concerning the properties of a material for which Poisson's ratio is 0.5, two possibilities exist: (1) If the shear modulus is zero; and (2) if the shear modulus is not zero. In the first case the medium resembles a perfect liquid; in the second case, the material is an incompressible solid. The authors further state that Equations (2), (3), and (4) are equations of equilibrium. These equations represent a partial statement of Hooke's law for an isotropic material.

Lateral Pressures Due to Surface Loads.—Under this heading the authors discuss the Boussinesq solution and certain subsidiary solutions obtained from Boussinesq's problem by integration. All these solutions satisfy the boundary conditions for the semi-infinite solid, and were intended by their originators to apply only to such a solid. The authors, however, propose to use these results to describe the pressure distributions against retaining walls and sheeting.

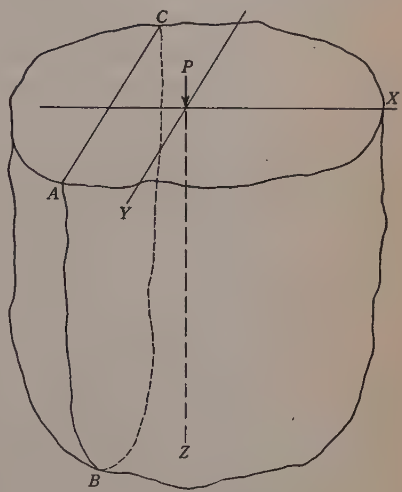

FIG. 11

The presence of an obstructing wall in the material completely changes the boundary conditions and, therefore, will change the state of stress in the solid. To visualize the causes of the change, consider, in the semi-infinite solid, a section, $A\ B\ C$ (Fig. 11), which is a vertical plane before the application of the surface load. Then, apply a load on the surface at some distance from the trace, $A\ C$, of the vertical plane in the surface. Plane $A\ B\ C$ then deforms into a curved surface and there will be certain stresses acting on this surface. Suppose, now, that the material to the left of $A\ B'\ C$ is removed. To retain the original state of stress in the material on the right side of $A\ B\ C$, the deformation of $A\ B\ C$ must be maintained by applying to this surface, from the left, the same distribution of stress that was originally supplied by the material removed. If either the stress or the strain on $A\ B\ C$ is changed, the Boussinesq distribution will no longer be maintained in the remaining material. If a wall is placed against $A\ B\ C$ instead of the stresses applied from the left, the deformation of Surface $A\ B\ C$ will be changed and, hence, the stress distribution both in the entire body and against $A\ B\ C$ will also change. The magnitude of these changes will depend upon the shape and elastic properties of the wall and its surface roughness. By the principle of superposition, the same conclusions will be reached if the wall were present from the beginning and the surface load applied later. That these considerations are not pure speculation is verified by the remarkable experiments of M. G. Spangler,[11]

[11] "Horizontal Pressures on Retaining Walls Due to Concentrated Surface Loads," by M. G. Spangler, *Bulletin No. 140*, Iowa Eng. Experiment Station, 1938.

Assoc. M. Am. Soc. C. E., which show that the actual pressures are about double those predicted by the authors.

It must be concluded, therefore, that Equations (12) to (27) do not apply to the problem under consideration. A sound study of the problem should include consideration of the interaction between the wall and the hypothetical elastic soil. A general solution of the problem involves difficulties which have not yet been overcome by students of elasticity. However, limiting cases have been treated and these throw light on the situation. For example, if the wall is relatively rigid in comparison with the soil, it may be assumed, as a first approximation, that the wall is perfectly rigid. Solutions obtained on this assumption [10] show rather good agreement with Professor Spangler's experiments. They do not, however, agree with the stress distribution depicted by the authors in Fig. 2. In fact, the authors' own proposals do not agree with Fig. 2.

"Rigid wall" solutions are also available for a variety of other boundary conditions. For example, the back face of the wall may be inclined, or two walls may form a right-angle corner. Consideration can also be given to the finite height of the walls and to the roughness or smoothness of some of the contact surfaces.

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[10] "Pressure Distribution on Retaining Walls," by Raymond D. Mindlin, Jun. Am. Soc. C. E., *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., Vol. III, 1936, p. 155.

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DISCUSSIONS

WIND FORCES ON A TALL BUILDING

Discussion

BY MESSRS. ROBINS FLEMING, F. P. SHEARWOOD, LYDIK S. JACOBSEN, FRANCIS L. CASTLEMAN, JR., J. B. WILBUR, R. D. SPELLMAN, DAVID A. MOLITOR, WALTER J. GRAY, AND K. L. DEBLOIS

ROBINS FLEMING,[14] ESQ. (by letter).[14a]—The forces treated in this paper are determined by actual measurement from observations extended over a period of five years. A wealth of data is given such as can be found nowhere else. It is a matter of congratulation to the structural engineering fraternity that the owners of the Empire State Building made it possible that such observations could be made. It is fortunate that so able a presentation of these observations as that given in the paper under discussion is available.

Professor Rathbun states that Table 1 justifies considerable detailed study. This is true not only of Table 1, but of the entire paper. Surprising varying wind velocities at near-by points are shown in Table 1. More striking still are the variations of pressure at different heights of the same building, as indicated in Table 2. This raises the question as to the allowable wind pressure that should be assumed in the design of a tall building. How shall it be distributed? Referring to the pressure obtained by Messrs. Dryden and Hill[3] from a model of the Empire State Building placed in a wind tunnel, the author writes: "A comparison of the pressures on the model and those on the building shows clearly that the natural wind movements are not at all like those in a wind tunnel." This is a striking sentence. How far-reaching is its application? Incidentally, a writer on this subject has stated[15] of a transmission tower in Belgium, erected for purposes of observation, that: "The installation will enable the relation between the aerodynamic coefficients of a model and a full-sized tower to be established, and will also enable work to be carried out as to the frequency of gusts."

NOTE.—The paper by J. Charles Rathbun, M. Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1938, by Messrs. David C. Coyle, and Clyde T. Morris.

[14] Civ. Engr. (*Retired*), New York, N. Y.

[14a] Received by the Secretary October 13, 1938.

[3] *Research Paper No. 525*, National Bureau of Standards.

[15] *Engineering*, May 6, 1938, p. 498.

Table 3, reporting the plumb-bob observations, is important. The writer notes that Professor Rathbun gives the maximum deflection of the eighty-sixth floor relative to the sixth floor (a distance of 974 ft), as 2.97 in. (see text following Equations (4)). The casual reader may have difficulty in reconciling this deflection with the sentences of ten pages previous (see heading "The Plumb-Bob") that:

"It is to be noted that the values in Table 3 do not indicate the maximum deflection of the building. They are only the mean position about which the building vibrates; for a maximum deflection one-half the amplitude of vibration should be added."

A few lines, or a page, could profitably have been added to the paper on this subject. The terms, "sway," "vibration," and "deflection," should be clearly defined. Where is the origin of co-ordinates taken?

In Table 5, the greatest amplitude of vibration recorded is from 0.5 to 5.2 in. in a north and south direction. The deflection, therefore, is $(5.2 - 0.5) \div 2 = 2.35$ in.

At this point attention is called to an opinion expressed[16] by Henry V. Spurr, M. Am. Soc. C. E. For a tower building 100 ft, center to center, of wall columns and 1 000 ft high, Mr. Spurr believes that a reasonable "yardstick" for a total deflection at the top is 0.001 h "which is about $\frac{1}{8}$ in. per story." The question may be raised: Will such a deflection give sufficient rigidity?

The literature of deflections is extremely meager. Few measurements have been made; or if they have been made they have not been published. Observations[17] made on two buildings in Chicago, Ill.—the 17-story Monadnock Block and the 14-story Pontiac Building—were reported in 1894:

"At the Monadnock Block the deflection was first observed by transits set in sheltered positions at points north and south of the north and south ends of the building and these observations were checked by observing the oscillation of plumb bobs suspended in the stairway shaft from the 16th floor. These two observations agreed very closely and show the vibration east and west from $\frac{1}{2}$ in. to $\frac{1}{4}$ in."

It should be said that the dead weight of the Monadnock Block would now be called excessive. This was the last skyscraper built with structural masonry walls. "The observations made on the Pontiac Building agreed quite closely with those on the Monadnock Block." No details are given. "The wind registered a little over 80 miles per hour."

In a monograph,[18] published in 1912, Cyrus A. Melick, M. Am. Soc. C. E., records measurements of the vibration of a bent in a 17-story building. The greatest indicated wind vibration was 1 in. in a period of about 4 sec. It will be remembered that the distance traveled during a vibration is twice the deflection from a normal position.

The invariable practice in the design of steel-frame tier buildings has been to proportion the steel frame to carry the entire wind load to the column bases.

[16] "Wind Bracing," by Henry V. Spurr, 1930, p. 27.

[17] *Engineering News*, March 1, 1894.

[18] "Stresses in Tall Buildings," Cyrus A. Melick, *Bulletin No. 8*, Coll. of Eng., Ohio State Univ., June, 1912.

Professor Morris[19] has stated that: "The stiffening effects of walls, partitions and floors should be neglected in calculating wind stresses. The entire wind load should be regarded as carried by the steel frame." The deflection calculated from the steel frame alone carrying wind pressure is much larger than the actual deflection. Mr. Molitor[7] computes the deflection of a 40-story building bent for a wind pressure of 15 lb per sq ft, to be 6.5 in. He adds a sentence, "If such a building in its completed state was to vary as much as 1 in. in the wind it might border on financial failure from the standpoint of occupancy."

The writer has calculated the deflection of the 20-story steel frame analyzed[20] by Wilbur M. Wilson and George A. Maney, Members, Am. Soc. C. E., in 1915, and found the deflection of the tops of the outer columns to be 3.91 in. This deflection was determined from Mohr's work equation, evaluating the integrals by a substitution formula given by Mr. Molitor.[21]

The process of calculating deflections in a multi-storied frame is long and tedious. For the Empire State Building it would be impracticable. Still, by an ingenious and elaborate method, Professor Rathbun arrives at the conclusion "that the structure is very close to four and one-half times as stiff as it would be if the steel frame alone held it in position" (see following Equation (1)).

The reason for the small observed deflections is discussed in Part II of the paper (under the heading, "Introduction"): "The capacity of a building to resist loads is not dependent upon the steel frame alone but upon the combination of steel and masonry as a unit." Mr. Molitor does not question any conclusions based on the theory of analysis as applied to a strictly engineering structure. "However," he writes,[22] "a steel building may be classed as an engineering structure only when the frame is bare; but when clothed with architectural coverings of concrete, stone and brick, with steel and concrete floors, tile partitions, etc., it becomes a composite structure, the nature of which cannot be appraised in terms of mathematics."

Mr. Jacob J. Creskoff[23] thinks that the present practice of restricting wind stresses to the steel frame is contrary to common sense: "A building resembles its frame only remotely, for the reason that the deflections of the horizontal members of the frame are limited by the slabs, and those of the vertical members are limited by the walls." He presents a "dynamic approach to design." The "vertical beams" are considered to be composed of reinforced concrete and structural steel.

Significant sentences in the paper by Professor Rathbun are: "Rigidity is of considerable importance inasmuch as it affects the popularity of a building with tenants. If the structure is too limber, the fact may be shown by cracks in the plaster and the movement may be noted in other ways causing lack of confidence

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[19] "Practical Design of Wind Bracing," by Clyde T. Morris, M. Am. Soc. C. E., *Proceedings*, Am. Inst. of Steel Construction, Inc., October, 1927.

[7] "Structural Engineering Problems," by David A. Molitor, M. Am. Soc. C. E., The Peters Co., Detroit, Mich., 1937, p. 64; also reproduced by the Bureau of Yards and Docks, U. S. Navy Dept., 1937.

[20] "Wind Stresses in the Steel Frames of Office Buildings," by Wilbur M. Wilson and George A. Maney, *Bulletin No. 80*, Eng. Experiment Station, Univ. of Illinois, June 7, 1915.

[21] "Structural Engineering Problems," by David A. Molitor, M. Am. Soc. C. E., The Peters Co., Detroit, Mich., 1937, p. 31.

[22] *Loc. cit.*, p. 63.

[23] "Earthquakes and Wind Design: Suggested Rationalization," by Jacob J. Creskoff, *Engineering News-Record*, Vol. 113, November 1, 1934, p. 533.

in the safety of the structure. On the other hand, if it is too stiff the period of vibration will be short enough to be noticeable by the tenants, thus producing the same lack of confidence" (see heading "Introduction" in Part II). The period of vibration is a function of the rigidity. Mr. Coyle believes that the necessity for stiffness depends on the occupancy. In 1929 he wrote[24]:

"The relation between type of occupancy and sensitiveness to motion is a very important one. In an office the occupants are awake, clothed, occupied and expected to be paid for their time. It is daylight and there are other people about to give a sense of security. In an apartment house one may find himself alone, in bed, and in the dark, with a storm howling outside. Moreover, if one is ever subject to disturbance of the organs of equilibrium, such a condition is most apt to recur after an evening. Such being the case it is inevitable that some of the tenants in an apartment tower will feel or imagine strange things, and tell them to their friends."

The writer recalls an instance where the scream of a girl in a dance hall led to the hall being closed. An investigation was ordered and the details of the steel frame were found to be reprehensible. The owners of the building were put to a heavy expense before the hall was allowed to be re-opened. Mr. Coyle thinks that safety and the approval of the Building Department are not all that need be considered in the design of a tall building; "there is also required a technique in dealing with the cause and cure of imaginary feelings on the part of the tenant."[25]

Of the nine "Conclusions" in the paper the writer is specially interested in Conclusion (7): "The cantilever and portal methods of design have neither been corroborated nor refuted by these data." Both these conventional methods have been severely criticized. Messrs. Wilson and Maney[20] pronounce the "portal" method so inaccurate that it should never be used; and declare that the "cantilever" method is "quite accurate in some cases but may give results that are seriously in error." A slope deflection is presented which they consider "very accurate." A shorter approximate method is then proposed. As previously mentioned the writer calculated the deflection of the steel frame from the data given in the text to be 3.91 in. In an apartment house such a deflection would certainly affect rental receipts.

The Third and Final Report of the British Steel Structures Research Committee—a volume of 599 pages—was published in 1936. The conventional methods of obtaining wind stresses, "such as the Portal and Cantilever methods," are condemned in this report, which also states that the steel frame is not alone in resisting wind forces. The "architectural clothing" has a share; and,

"Floors are not the only clothing likely to affect the stresses in the steel framework. The fire-resisting stanchions and the external walls will play their part. * * * These tests have shown that the effect of clothing cannot be ignored. * * * In a large class of steel-framed buildings the floors and walls in conjunction so brace the framework that appreciable sway due to wind is prevented and the steel work is thus relieved of the responsibility of resisting wind shear."

[24] "Mushroom Skyscrapers," by David C. Coyle, M. Am. Soc. C. E., *The American Architect*, Vol. CXXXV, p. 289, June 20, 1929.

[25] "Testing the Strength of Skyscrapers," by David C. Coyle, M. Am. Soc. C. E., *Record and Guide*, Vol. 127, February 7, 1931, p. 12.

In view of the foregoing and similar statements it is pertinent to ask why the conventional methods of obtaining wind stresses do not give ample security. Are they not as nearly correct as the elaborate and sometimes impractical methods that have been proposed? Mr. R. Gray, a British engineer, in a paper, "Wind Stresses in Tall Buildings,"[26] gives three methods for obtaining moments, shears, and thrusts, of which the "cantilever" and the "portal" are the second and third. Professor Rathbun considers that proposed methods of obtaining so-called "exact" results are unworkable except in the simplest cases. "Moreover," he states, "it is doubtful whether they are much nearer the truth than the results given by the common methods, since the influence of the walls and floors, though quite considerable, is quite incalculable."

In connection with Conclusion (9), it is interesting to note that the horizontal pull at the eighty-sixth floor, necessary to produce a deflection of 2.97 in., is equivalent to that produced by a uniform load of 13.4 lb per sq ft over the south face of the building (see following Equations (4)).

It may be of historical interest that in order to deny the truth of current reported deflections a communication, signed "Empire State Incorporated," appeared in the *New York Sun* of July 10, 1931. The communication reads:

> "Please be informed that while no gale strong enough to permit measurements to be made has ever been furnished by the elements in New York City, it is a firm belief of the architects and the engineers who built Empire State that the possible swaying of the building in a 220 mile wind would be absolutely negligible. The highest measurement made so far indicates a sway of less than one-tenth of an inch."

Paul Starrett, of Starrett Brothers and Eken, the firm that built the Empire State Building, writes in 1938[27]: "The sway of the Empire State Building has been measured in strong wind-storms and the greatest variation of the mooring mast from the perpendicular was found to be two and a half inches on either side."

The highest velocity recorded in the tables of Professor Rathbun's paper is 90 miles per hr although a velocity of 102 miles per hr is mentioned in the text (see heading, "Velocity of the Wind").

The New York City Building Code for structures higher than 100 ft calls for an assumed wind pressure of 20 lb per sq ft of exposed surface from the top down to the 100-ft level. By an Amendment that became effective March 24, 1930, the requirement in the then existing Code was changed from 30 lb to the present 20 lb. The Empire State Building was formally opened to the public on May 1, 1931. The late H. G. Balcom, M. Am. Soc. C. E., reported[28] that:

> "In wind calculation, the floor construction was assumed to act as a rigid horizontal plate, which distributed the wind pressure to the various bents in the ratio of their resistance. Overturning wind stresses were figured by the cantilever method. The resistance was based on the relative moments of inertia of the different bents; that is, the overturning unit pressure, positive or negative, taken by each column was proportioned to its distance from the center of gravity of the bents."

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[26] *The Structural Engineer*, Vol. XV, May, 1937, p. 186.

[27] "Changing the Skyline: An Autobiography," by Paul Starrett, 1938.

[28] "Wind Calculations," by H. G. Balcom, *Civil Engineering*, Vol. 1, May, 1931, p. 700.

The paper by Professor Rathbun is an apt rejoinder to a hope expressed by Mr. Balcom in the last sentence of his article,[28] " * * * it is, therefore, greatly to be hoped that the building may be noted, not only for its height and majestic beauty, but also as a means of promoting engineering knowledge."

F. P. Shearwood,[29] M. Am. Soc. C. E. (by letter).[29a]—A fascinating problem is treated in this paper in that it deals with one of the major factors in the intricate problem of designing the frame of a high building, safely and economically. The records indicate that the wind forces acting on a high building are not nearly as severe as most building codes specify, and that the effect of gusts does not extend to a great depth. The paper indicates that the resulting forces on the sides of the building are very erratic and may create torsion in the structure.

A most useful and interesting part of the paper is the reference to the great initial resistance of the masonry. Some of this resistance was found to be plastic; that is, the deflection did not recover. This would indicate that movement had occurred between some parts of the material, or, that some permanent stretch or compression had taken place. Masonry has practically no ductility and, therefore, no resistance that is not elastic, except the friction between its particles. The steel frame is wholly elastic for loads far greater than those applied during the tests, with the exception of the possibility that the riveted joints can slip in their holes; but this is improbable at the small wind loads recorded during the tests.

If, therefore, the masonry has practically no ductility and the steel frame connections have not slipped, the plastic deflection of the building must be due to the fracturing of some of the elastic resistance of the masonry, and then to its overcoming the frictional resistance to its slipping on itself or on the steelwork.

A tremendous amount of labor, discussion, and thought has been expended on devising theories for calculating the exact distribution of the resistance to the wind forces on these steel frames, but each investigator disregards a great many actual facts which must modify the theories to such an extent that they probably do not agree, even approximately, with actual results.

To devise an exact solution of theory for such an indeterminate stress problem, and one which is based on very doubtful assumptions, seems like aiming at needless refinement. The resisting frame has many paths of resistance; it is composed of a material that has considerable ductility and elasticity of about twice the amount that is ordinarily utilized in design. Structural designing is restricted and influenced far too much by arbitrary application of unit stresses and other limitations, which are applied impartially to the computed stresses in members. Some of these members must resist the stress without assistance from adjacent members, whereas others, when overstrained, will automatically bring other paths into increasing assistance.

Roughly stated, in designing the frame of a high building the present custom is to consider it on the following bases:

(1) As composed of an articulated frame for calculating the vertical loads;

[29] Cons. Engr., Dominion Bridge Co., Ltd., Montreal, Que., Canada.

[29a] Received by the Secretary October 13, 1938.

(2) As composed of independent portal bents with rigid joints for calculating the horizontal forces;

(3) That the wind force is a uniformly applied load; and,

(4) That the steelwork is assembled and fabricated without having initial strain or slackness, and that it is wholly and perfectly elastic.

In actual practice not one of these conditions is quite correct; but it is assumed arbitrarily that they are all correct, and on that basis the stresses are calculated and the material is strictly porportioned.

The important questions raised by the tests in this paper and which require further investigation are: (*a*) Whether the great proportion of resistance that the masonry exerts against the wind forces will be maintained when much higher winds occur; and (*b*) whether, or how much of, this resistance will be fractured and, therefore, destroyed before the wind forces reach their maximum intensities. There might be much insurance against partial damage to the masonry, etc., should severe winds occur, by utilizing more fully the potential stiffening effect of the masonry, anchoring it firmly to the steelwork, and even reinforcing the walls.

These tests show how very problematical are the assumed forces and resistances which are commonly used as the bases of the theories and calculations for designing steel building frames. They indicate that a building should be treated more as a monolithic whole rather than just as an independent frame composed of supports and bents. It is especially important to guard against details that have planes of weakness or relatively excessive or sudden changes in stiffness, where strains will be dangerously concentrated when meeting the general deflections. Finally, they suggest that the stresses given by the conventional theories (which endeavor to distribute, exactly, the indeterminate stresses induced by indeterminate forces) should be used only as a guide, and that engineering judgment and experience should be given greater latitude.

Lydik S. Jacobsen,[30] Esq. (by letter).[30a]—A series of interesting tests is reported in this paper, which contains information of great value to the student of structural engineering. From the point of view of fundamental knowledge of the dynamic behavior of a building, the author's observations of the longer of the fundamental transverse periods of the Empire State Building is especially illuminating and is in good agreement with numerous observations made on buildings in California.

Equation (1), in Part II, appears to be somewhat misleading since it applies only to ratios. In other words, T is not the period of vibration; it is only the ratio of what Professor Rathbun calls homologous times. Similarly, W is not the weight of the building or of the model; it is only the ratio of homologous weights or masses, etc. Aside from this obvious over-sight, it is extremely interesting to note that the model period converted to the prototype dimensions becomes 17.8 sec whereas the observed building period is 8.25 sec. Since the influence of the foundation on the fundamental period of the actual building is not well known quantitatively, it was probably not considered in the model, so

[30] Prof., Mech. Eng., Stanford Univ., Stanford University, Calif.

[30a] Received by the Secretary October 17, 1938.

that the 17.8-sec period obtained by the model must have been based on the assumption of an infinitely rigid foundation. With this in mind, it is obvious that the stiffening effect of the masonry must be somewhat greater than the author's estimate of 350 per cent. Data obtained by the United States Coast and Geodetic Survey[31] on buildings at different stages of construction, support the view that the masonry walls may stiffen the structure as much as ten times the steel frame stiffness.

It is also of great interest to note that between 1932 and 1934 the observed period of the building decreased from 8.38 sec to 8.14 sec. One wonders if the present period of 8.25 sec indicates an actual lengthening since 1934, or if the methods of measuring the periods are really of three-place accuracy. The author's explanation that the shortening of the period is probably due to an adjustment of the several parts of the structure during severe storms is quite plausible, especially if the least known part of the structure—the ground on which it stands—is included. It is suggested that a time record of accurate determinations of periods be continued.

The fact that the building does not have a unique position of static equilibrium, as indicated by the collimator observations, is significant. Undoubtedly, it must be given the author's interpretation as being due to plastic action, within the structure itself, although even in that case the ground effect, if it is non-uniform over the foundations, may account for part of the wandering of the middle position of swing in still weather.

Considering the well-known fact that wind structure, especially during storms, is a very complicated phenomenon,[32] it is not strange that the simultaneous readings of wind velocities obtained at the four stations in the vicinity of the Empire State Building differ greatly. One wonders if the four lengthy pages of Table 1 really justify "considerable detailed study."

Conclusion (1) relating to the complicated and irregular distribution of wind pressure on a building is amply warranted by even a cursory glance at Table 2. It is further corroborated by common-sense observations as well as by the aforementioned technical studies.[32]

The author's derivation of formulas, beginning with Equation (5) and ending with Equation (20), is extremely involved considering that the result (Equation (20)) can be written down almost *a priori* and that Equation (19) simply shows, in integral form, the sum of some of the potential energies divided by the kinetic. Since the author agrees with other investigators about the great stiffening effect of the masonry walls, one is surprised to find that the theory involves only the usual "static" framework consisting of columns and girders, and that the more important members—the walls—are entirely neglected. The writer suggests that, if an explicit formula for the fundamental transverse period is to be developed, a method involving an equivalent cross-section of a homogeneous, although far from uniform, beam may lead to simpler and more accurate results. However, the step-by-step method of calculating the period

[31] Preliminary Repts. of U. S. Coast and Geodetic Survey's California Seismological Program.

[32] "Wind Structure in Winter Storms," by R. H. Sherlock, M. Am. Soc. C. E., and M. B. Stout, *Journal of the Aeronautical Sciences*, December, 1937.

is patently indicated for a building of as complicated an outline as the Empire State Building.

In spite of this seemingly critical discussion of the paper, the writer wishes to record his great appreciation of this interesting and important contribution to the knowledge of the dynamic behavior of buildings.

Francis L. Castleman, Jr.,[33] Assoc. M. Am. Soc. C. E. (by letter).[33a]—The Technical Research Committee of the American Institute of Steel Construction, and the owners, architects, consulting engineers, and contractors for the Empire State Building are to be congratulated on the foresight shown in equipping this building in such a manner that pertinent data on wind and its effect could be recorded. Professor Rathbun has presented an interesting and worthy paper describing these data and it should stimulate interest in their proper evaluation. Quantitatively, many more large-scale experiments appear necessary before any definite interpretations or conclusions can be drawn; but, qualitatively, the data submitted tend to clarify and sustain some common conceptions on this intricate subject.

Constructive criticism would point to the relative brevity of the paper. As the work extended over a period of five years, considerably more explanatory data might have been inserted at the expense of brevity. On first reading, the paper appears somewhat voluminous, but on detail study the writer thought certain parts meager and fragmentary.

Although this building is monumental in that it is the tallest ever built, there is a particular property it has in common with many other buildings; namely, the ratio of its maximum height to its least width is less than 8. A maximum height of 1 049 ft and a least width of 134 ft are assumed. This feature is emphasized in the hope of encouraging like studies on buildings with a similar ratio, but of not such great height. The writer knows of several buildings exceeding this ratio.

A theoretically correct mathematical analysis, taking all factors and their combined effect into consideration, would be difficult for a structure of this nature. The bracing in the central bays around the elevator shafts, plus the shallow "knuckle" wind-bracing, constitute a combined system which, when considered as a unit, forms a complicated but effective arrangement. This should be borne in mind when interpreting much of the data submitted.

As stated in the paper, the plastic action of the surrounding material has a marked effect in reducing deflection and vibration at ordinary wind loads—even at high wind loads. In the present state of knowledge, however, it is doubtful whether these materials should be counted at extreme loadings, as their beneficial effects under such conditions are reduced. Extreme loadings occur perhaps only two or three times in the life of such a structure; but on such occasions it must be able to sustain them adequately. Mr. Charles B. Driscoll[34] has commented interestingly on his esoteric experiences during a recent storm as follows: "Yes, the tall towers swayed. Some of them rather

[33] Assoc. Prof., Structural Eng., Vanderbilt Univ., Nashville, Tenn.

[33a] Received by the Secretary October 18, 1938.

[34] "New York Day by Day," McNaught Syndicate Inc., *Nashville Banner*, October 5, 1938, p. 9.

sickeningly. There were plenty of towers in which office employees became seasick during that stormy afternoon. * * * A good many were excused from work." These remarks from a layman should be convincing in themselves.

Although the Empire State Building is much higher than its neighbors, the latter will have a considerable effect on the resulting action of the wind. Until wind-tunnel methods are modified to make allowance for this effect, as well as other factors, experimental data based on such methods can be of little help.

The wind records given in Table 1 are particularly enlightening and, as the author states, justify considerable detailed study. Twenty-three items in this table show wind velocities of 50 miles per hr, or more. Using the well-known relation, $p = 0.003\ V^2$ for a 50-mile wind, $p = 7.5$ lb per sq ft; for a 70-mile wind, $p = 14.7$ lb per sq ft; and, for a 102-mile wind, $p = 31.2$ lb per sq ft. It is assumed in the foregoing that true and not indicated velocities are dealt with. (Lack of this distinction is a source of continual annoyance and error in many published data.) In the storm that swept over a part of New England in September, 1938, an Associated Press Dispatch of September 23 stated that wind gusts of approximately 173 and 186 miles per hr were recorded at the Harvard Meteorological Observatory. In design, usually, the average speed over a few minutes is considered satisfactory, but there is a school of thought which maintains that extreme velocities should be used. From the data supplied, and considering its erratic nature, it would seem that 20 lb per sq ft taken by the frame alone is not over-conservative in the vicinity of New York City. Table 1 indicates that the pressure varies with the height (which has long been known), but that there appears to be no discoverable law of variation for such a densely built area. Many building codes allow for this condition by varying the pressure approximately directly as the height, with certain fixed limitations. Considering the present state of knowledge this may be a wasted effort complicating the work of the structural engineer. Until more data are forthcoming, the best that can be done is to use a sum total effect that approximates the actual conditions in the light of present-day knowledge, in an endeavor to produce an adequate structure from the standpoint of human safety and comfort, rather than from that of cost. This is not as easy as it sounds.

From a meteorological viewpoint three weather bureaus on Manhattan Island may be sufficient, but from the standpoint of the structural engineer, or rather the data that should be furnished to him by the meteorologist, this number would appear deficient.

Fig. 8 seems to indicate that on the windward side of the building the air currents tend down, whereas on the leeward side the tendency is upward. As Fig. 8 is indicative of a single observation it would be interesting to know if this is true in general, or even approximately so. The manometer readings exhibit qualitatively the complex action of the wind and the great variations in pressure both positive and negative. However, as Professor Rathbun states, they are not to be taken as giving a true picture of the actual forces acting on the building. Although the influence of temperature on deflection has been shown to be negligible on this building, the writer would be wary

as to accepting this in principle. With the rapid developments made in light-weight covering, and considering similar future advances, this might not always be true, by any means.

The author has made computations assuming the structure to be a cantilever, applying the well-known flexure formula, $s = \frac{M\,c}{I}$. As he states, this presupposes that the twenty-fourth and twenty-fifth floors remain planes after elastic action from wind has occurred. With the use of the "knuckle" wind connection, this is questionable. Although this type of connection has strength, it is relatively deficient in stiffness. Such computations should be accepted with reservations. Columns (13) and (14), Table 6, would seem to support this view. Furthermore, as one of the studies contemplated was, "a check, or refutation, of the basic principles of the portal and cantilever methods of analysis," the writer cannot help but wonder why the portal method was not at least touched upon for comparative data. However, it is only fair to state that for this building, with a double type of wind-bracing, any method, theoretical or otherwise, will be difficult to apply as the division of wind loading between the two systems leads to complicating ramifications.

It is stated that the plasticity of the surrounding materials has a dampening effect on the vibrations, but does not alter the period. This is true but does not tell the entire story as to ultimate possibilities which might occur. Although the possibility is remote, the disturbing wind forces might sometimes approximate a periodic nature for a short interval setting up a tendency toward a forced vibration. If the frequency of the disturbing forces is seriously different from the natural frequency the resulting vibrations will be small. If by chance the two frequencies are equal, or only approximately so, the amplitude is increased enormously.

In computing the equivalent uniform load by means of the model and graphical integration a minimum reading of 3.26 in., as given in Table 3, under Item No. 115, might be a more conservative value to use in the absence of detailed explanation. The deflection would then be $\Delta = 3.25$ in. The ratio of this deflection to that produced by a 1-lb load on the model would be $\frac{3.25}{0.0488} = 66.6$. The new horizontal pull at the eighty-sixth floor necessary to produce this deflection would be $P = \frac{66.6 \times 50\,530\,000}{(60.484)^2} = 919\,900$ lb. From the curves of Fig. 18 the same deflection would be produced by a uniform load over the north face of $\frac{919\,900}{64\,600} = 14.24$ lb per sq ft (including negative as well as positive pressure). As Professor Rathbun states, such a value should be considered with caution.

It would appear that Equation (18) lacks one term. The direct stress, s_1, in a beam at any floor level will be proportional to the difference of the shear below and above the level considered. Therefore:

$$s_1 = \frac{p_1}{q_1}\left(\frac{4\,\pi^2}{T^2}\right)\int_{x_1}^{h} m\,y\,dx \qquad (21)$$

in which p_1 and q_1 are abstract dimensionless numbers. The energy in a beam under axial load is, $W_b = \frac{s_1^2 L}{2 A E}$, or,

$$W_b = \frac{L}{2 A E} \frac{16 \pi^4}{T^4} \frac{p_1^2}{q_1^2} \left[\int_{x_1}^{h} m\, y\, dx \right]^2 \ldots\ldots\ldots\ldots (22a)$$

and for all beams it will be,

$$\sum W_b = \frac{16 \pi^4}{2 T^4} \sum \frac{p_1^2}{q_1^2} \frac{L}{A E} \left[\int_{x_1}^{h} m\, y\, dx \right]^2 \ldots\ldots\ldots\ldots (22b)$$

Using capital letters for the dimensionless terms this will reduce (after multiplying through by T^4) to,

$$T^4 \sum W_b = G \sum \left(\frac{L}{A E} \right) \left[\int_{x_1}^{h} m\, y\, dx \right]^2 \ldots\ldots\ldots\ldots (23)$$

The term on the right-hand side of Equation (23) should appear in the right-hand side of Equation (18). If it is admissible to neglect the third term in Equation (18), the term developed in Equation (23) can probably be neglected. For tower buildings the axial distortions of the columns are considerable and it is a question as to whether the third term in Equation (18) should be neglected. If it were necessary to consider it, much of the work of the author's paper might be invalidated as the area of the columns and beams of the model does not bear the proper similitude to the building itself. More information concerning the building and model would appear necessary to evaluate properly the weight that this term carries.

J. B. WILBUR,[35] ASSOC. M. AM. SOC. C. E. (by letter).[35a]—The subject of stresses in building frames due to lateral forces is as complicated as it is important. It is a problem that may be broken down into a large number of subdivisions, each one of which is so complicated as to discourage the hope of satisfactory solution without extended studies and research. It is only by the constant investigation of the various problems involved that satisfactory methods for the design of the frames of tall buildings may eventually be discovered. Professor Rathbun's data and discussions are especially helpful inasmuch as they are based on the action of an actual building.

The stiffening effect of the masonry of a building, as Professor Rathbun states, is a problem of considerable importance. Although it is undoubtedly important under most conditions, it is probably true that the stiffening effect of the masonry becomes of less importance as the loads increase. In the First Progress Report of the Committee on Steel of the Structural Division (Sub-Committee No. 31 on Wind Bracing in Steel Buildings),[36] the suggestion was made that judgment as to the stiffness of a building be based on a deflection index. This index was defined as the maximum deflection at the top of a

[35] Assoc. Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

[35a] Received by the Secretary October 22, 1938.

[36] *Civil Engineering*, March, 1931, p. 478.

frame (under maximum wind load and assuming the frame to carry the entire wind load) divided by the height of the frame. It was recognized that under normal conditions the deflection index corresponds to some multiple of the deflections that will actually occur, but the deflection index was suggested, nevertheless, as a convenient standard by which to judge the probable stiffness of a building. Although the use of the deflection index is open to criticism because of the fact that so much is left to judgment, in respect to the stiffening effect of masonry, the writer knows of no better practical approach to the problem. Some of Professor Rathbun's data should be of real value in applying the judgment factor in the use of the deflection index.

When one wishes to make use of the deflection index, the question arises as to how to compute the deflection of the top of a building frame under the action of lateral loads. If a complete analysis of the frame has been made, this is easily accomplished by considering the action of one column stack by itself, and applying the moment-area method. It is often desirable, however, to estimate the deflection index without a complete stress analysis for the frame. Investigations by the writer have shown that deflection computations based on moments as analyzed by easily applied methods, such as the portal and cantilever methods, are likely to be seriously in error. In the belief that an approximate slope deflection solution might be used to advantage in this problem, an investigation was conducted, in 1933, under the supervision of the writer, by William Niessen, Jun. Am. Soc. C. E., and Mr. R. J. Stoddard.[37] The procedures followed were similar to those suggested by the writer in a previous publication dealing with an approximate slope deflection determination of moments in building frames due to lateral forces.[38] The method of distributing wind loads to the bents of a building which resulted from this investigation has been presented elsewhere,[39] but an interesting result of this research, which was not cited at that time, consisted of a method for determining the deflection of the top of a building bent, which is not only easily applied, but which is reasonably accurate. In the Appendix of that paper,[39] it was shown that, approximately,

$$\delta_o = \frac{H_o\, h_o}{24\, E}\left(\frac{2}{\sum K_{co}} + \frac{1}{\sum K_{ga}} + \frac{1}{\sum K_{gb}}\right) \qquad (24)$$

In Equation (24), all symbols apply to one bent only, as follows: δ_o = the increment to horizontal deflection occurring in a given story which is denoted by the subscript, o; H_o = total horizontal shear on the given story; h_o = height of the given story; E = modulus of elasticity of the material of the bent; $K = \frac{E\,I}{L}$ for a member, in which I and L are moment of inertia and length, respectively, for the member; $\sum K_{co}$ = the sum of the K-values for all the

[37] "A Theoretical Investigation to Determine the Proportion of Wind Pressure Carried by the Various Bents of High Buildings," by W. Niessen and R. J. Stoddard. Submitted, in May, 1933, in partial fulfillment of the requirement for the degree of Master of Science.

[38] "A New Method for Analyzing Stresses due to Lateral Forces in Building Frames," by John B. Wilbur, Assoc. M. Am. Soc. C. E., *Journal*, Boston Soc. of Civ. Engrs., January, 1934.

[39] "Distribution of Wind Loads to the Bents of a Building," by John B. Wilbur, *Journal*, Boston Soc. of Civ. Engrs., October, 1935.

columns of the given story; $\sum K_{ga}$ = the sum of the K-values for all the girders of the floor directly above the story under consideration; and, $\sum K_{gb}$ = the sum of the K-values for all the girders of the floor directly below the story under consideration.

To obtain the deflection of the top of a bent, Equation (24) is summed up for all the stories of the bent:

$$\delta_{top} = \frac{1}{24\,E} \sum_{\text{basement}}^{\text{roof}} \left[H_o\, h_o^2 \left(\frac{2}{\sum K_{co}} + \frac{1}{\sum K_{ga}} + \frac{1}{\sum K_{gb}} \right) \right] \ldots\ldots (25)$$

In Equation (25), $\frac{1}{\sum K_{gb}}$ equals zero for the basement if the columns are considered as fixed at their base, since this condition is equivalent to girders of infinite stiffness.

Equation (25) may be evaluated directly for any given bent, if the loads acting on the bent and the make-up of the members are known. Comparing the deflections of the tops of nine different bents of varying arrangements and sizes as computed by Equation (25), with corresponding deflections based on so-called exact solutions by the slope deflection method, it was found that the average error resulting from the approximate nature of Equation (25) was 5%, whereas the maximum error was 13.5 per cent. In view of the fact that deflection indices based on these deflections would be used only as a guide to judgment in estimating probable building stiffness, the accuracy obtained is undoubtedly sufficient for practical purposes.

R. D. Spellman,[40] Assoc. M. Am. Soc. C. E. (by letter).[40a]—Regarding the effect of masonry upon the rigidity and strength of a building against wind pressure, as treated in the paper by Professor Rathbun, the writer would like to call attention to some observations made in Miami, Fla., after the 1926 hurricane. All the larger buildings in Miami, and especially those few which were structurally damaged by the hurricane, were inspected by the writer with the aid of the plans and specifications.

In most of the buildings damaged some kind of wind-bracing was included in the structural steel frame. Several were quite thoroughly designed, and, in others, the wind-bracing was covered by calling for top and bottom angles in the specifications. However, there was no apparent relation between the amount of the wind-bracing provided and the damage done to the different buildings. Some of the better designed buildings were damaged to a greater extent than others with practically no wind-bracing in the structural steel frame. The writer suspected that this inconsistency was due to the stiffening effect of the masonry. Calculations were made for each building to determine the unit shear on all masonry walls caused by the total wind load on the building above the floor where the greatest damage occurred. It was discovered that the damage was in close ratio to these unit shears.

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[40] Chf. Engr., Industrial Bldg. Dept., Rust Eng. Co., Pittsburgh, Pa.

[40a] Received by the Secretary October 28, 1938.

When the calculations were made, a value of 60 lb per sq ft was assumed as the maximum pressure reached during the hurricane. The unit shears varied from about 40 lb per sq in. to 110 lb per sq in. in the Meyer Kiser Building, which failed completely near the sixth floor. Another building which had a unit shear of 75 lb per sq in. was damaged quite badly. As the shears were reduced to nearly 50 lb, the damage was quite small, and one building with practically no wind-bracing, but with a unit shear of about 40 lb, had only a few cracks in the plaster on the first floor. This building, however, was better protected by other buildings than most of those damaged severely. All the foregoing calculations were made hurriedly and no specific value should be attached to them, except to note that the damage was in proportion to the unit shears on the masonry walls of the buildings, and perhaps to show about where damage may be expected to begin. All the aforementioned buildings were between fifteen and thirty stories high.

Most buildings in Miami, and especially those referred to herein, are constructed with light masonry walls; buildings of the same class in New York City, or other parts of the United States, would have 50% to 100% more cross-sectional area of masonry walls.

It is not the writer's intent to leave the impression that damage was caused to some of these buildings in spite of the fact that they were provided with adequate wind-bracing in the structural frame. Had they been properly designed, it is certain that no damage would have occurred; for example, the Congress Building had been carefully designed for a wind pressure of 25 lb per sq in. and, at the time of the hurricane, had reached that stage in the construction at which all the exterior walls were completed but few of the interior walls and partitions had been installed, giving it the maximum wind load with the minimum masonry resistance.

Commenting on Conclusion (8) of the paper, it would seem that in some instances, depending on the proportions of the building and the quantity and distribution of masonry therein, a substantial saving could be attained safely by omitting the wind-bracing in the structural frame. This might apply to the entire building, or to the upper parts of most buildings. Perhaps the point at which this wind-bracing might be omitted could be determined at a certain unit shear on the masonry walls. When the unit shear exceeded this value, the entire load would then be taken by the structural steel frame, and the masonry disregarded both for strength and stiffness.

DAVID A. MOLITOR,[41] M. AM. SOC. C. E. (by letter).[41a]—The most complete answer hitherto published, relative to the actual functioning of the steel frame of a tall building, is contained in this paper. The author's findings, based on the observations and measurements made during high winds, at various times, over a period of years, constitute a valuable contribution to knowledge.

The assumptions which have served as criteria in formulating methods for the design of tall buildings, crude as they are, have generally given very satisfactory results in producing tenantable structures. However, these achievements have been attained by a process of gradual evolution, in which the

[41] Cons. Engr., Harlingen, Tex.

[41a] Received by the Secretary November 15, 1938.

progressive steps from smaller to taller buildings were verified by actual structures, each time after venturing a little higher than in previous cases; and now the profession learns from a large full-sized model (the Empire State Building) that the assumed design unit stresses for resisting wind loads are about three times those actually occurring in the steel frame when subjected to such wind loads; and, that the maximum deflection at the top of the cantilever is only about one-third as much as would be expected if the steel frame were fully loaded and stressed according to the design assumptions.

Some years ago the writer attempted to appraise the relative stiffness of a building frame as affected by the contributing stiffness of the architectural clothing,[7] and, strangely enough, arrived at a conclusion which is now found to check remarkably well with the results of the actual measurements obtained on the Empire State Building. These findings prompted the writer to express the opinion that all refinements pertaining to wind stress analyses in building frames border on wasted energy, and that rational design methods only could lead to satisfactory and economic construction.

The deflection of a 40-story building frame was computed to be 6.5 in., on the assumption that the frame carried the entire wind load of 15 lb per sq ft on the basis of 24 000-lb unit stress. The probable deflection of this building would not exceed 1.5 in., leading to the conclusion that buildings thus designed would deflect three to four times as much as they actually do. In other words, the architectural clothing, and not the steel frame alone, is responsible for the rigidity found by experience to be manifested in present-day tall buildings.

Were it not for the inherent rigidity of the architectural clothing of the steel frame, current design assumptions would be seriously deficient. Fortunately, the combination of a flexible steel frame, designed for certain wind loads and unit stresses, with a comparatively rigid fill material such as concrete, does produce tall buildings with tenantable rigidity. There is one mitigating circumstance which makes vibratory disturbances more tolerable, and that is the period of the vibration which may vary from 5 to 10 sec.

However, rigidity constitutes an important asset to the owner of a building, and the degree which the steel frame alone contributes to this factor, despite the severe load assumptions, is, in fact, relatively insufficient to supply the necessity.

In view of the foregoing and the substantial agreement with the findings presented in the paper, it would be very risky to adopt any changes in the present methods of design, or to choose different kinds of steel such as silicon steel, with higher unit stresses which would result in more flexible frames, thus incurring the danger of introducing new difficulties which might easily result in untenantable buildings. If such an error of judgment were committed in a design, the result might produce failure of the financial undertaking.

It would be highly desirable to have similar measurements made on other new buildings so as to add more of the valuable data to present knowledge on this subject. However, the irrefutable facts presented in this paper should put a quietus on most of the theoretic speculations indulged in by writers on

[7] "Structural Engineering Problems," by David A. Molitor, The Peters Co., Detroit, Mich., 1937, p. 64; also, reproduced by the Bureau of Yards and Docks, U. S. Navy Dept., 1937.

this subject, and designers should continue to use the simple approximate methods which have given such good results in practice.

WALTER J. GRAY,[42] JUN. AM. SOC. C. E. (by letter).[42a]—Adding to the interesting record of the variations of wind action on the Empire State Building, as reported by Professor Rathbun, is the observed fog characteristic shown in Fig. 23. As indicated, the fog formed a "wake" on the north face of the building, contracting to a neck at a distance of about 100 ft from the structure and then suddenly spreading again to dissipate itself in the direction of the wind. The phenomenon was visible from the street level since the concentration of fog in the "wake" caused it to appear darker than the surrounding fog body.

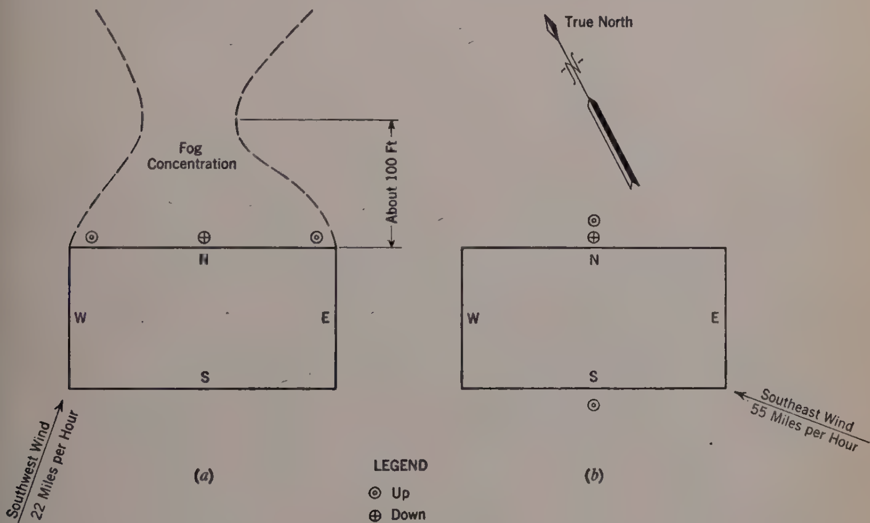

FIG. 23

This "wake," similar in character to the phenomena explained in texts on aerodynamics, substantiates the existence of a negative pressure on the far side of the structure, which characteristic is indicated by the manometer tube readings in Fig. 10 of the original paper. At the time of the observation, air currents close to the building indicated only vertical movements on the north face. The observation was made at a time of low wind velocity as shown by the anemometer at the top of the building. Subsequent observations in a heavy fog failed to indicate that the visible phenomenon occurred at higher wind velocities. The observations shown in Fig. 23(*b*) were taken under fog conditions similar to those of Fig. 23(*a*), but no "wake" was seen in the case of the higher velocity. It would appear that although a negative pressure exists at higher velocities, its visible evidence is destroyed by the gusty character of the wind.

[42] Topographical Draftsman, Bureau of Sewers and Highways, Office of Pres., Borough of Manhattan, New York, N. Y.

[42a] Received by the Secretary December 5, 1938.

K. L. DeBlois,[43] Assoc. M. Am. Soc. C. E. (by letter).[43a]—The vibration tests on the model as described under the heading, "Description of the Models," provide an opportunity to measure the damping. The shape of the curves shown in Fig. 20 indicates that damping is due to elastic or internal friction of the material, and, being proportional to velocity, the damping has the effect of reducing the oscillations in geometrical progression.[44]

The equation of the curve connecting the extremities of the oscillations in Fig. 20 is given by,

$$\alpha_r = \alpha_0\, e^{-2\pi\delta t} \qquad \text{(26)}$$

in which α_0 is the initial amplitude; α_r is any succeeding amplitude; t = elapsed time; and, δ = damping coefficient.

From inspection of Equation (26) it is evident that any convenient scale may be used to measure α_0 and α_r. Using Fig. 20(*b*): $\alpha_0 = 0.27$ in.; $\alpha_r = 0.01$ in. after 31 vibrations; $t = 4.0$ sec; and, T = period $= \dfrac{4}{31} = 0.129$. Substituting these values in Equation (26): $0.01 = 0.27\, e^{-2\pi\delta 4.0} = 0.27\, e^{-25.13\delta}$; from which, by logarithms, $\delta = 0.131$, and the damping factor, $e^{-2\pi\delta T} = 0.90$. This damping factor expresses the rate at which the vibrations tend to disappear; that is, the amplitude of any vibration is 0.90 of the amplitude of the preceding one on the same side of the axis.

Applying the same procedure to the building, which has an observed period of 8.25 sec, and assuming that the ratio of the amplitudes after 31 vibrations is the same as in the model: $t = 8.25\,(31) = 255.5$ sec; and, $\delta = 0.002$. The damping factor $e^{-2\pi\delta T}$ is 0.90, the same as for the model.

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[43] Associate Highway Bridge Engr., U. S. Bureau of Public Roads, San Francisco, Calif.

[43a] Received by the Secretary December 6, 1938.

[44] *Area*, Vol. 37, No. 380, October, 1935, p. 28.

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DISCUSSIONS

TRANSPORTATION OF SAND AND GRAVEL IN A FOUR-INCH PIPE

Discussion

BY MORROUGH P. O'BRIEN, M. AM. SOC. C. E., AND R. G. FOLSOM, ESQ.

MORROUGH P. O'BRIEN,[20] M. AM. SOC. C. E., AND R. G. FOLSOM,[21] ESQ. (by letter).[21a]—The writers have published[22] a paper on the transportation of sand in pipe lines, which presented experimental results on a number of sands. They have correlated these results with previously published data on materials ranging in specific gravity from 2.6 to 3.5 and median diameters from 0.0005 to 0.067 in. As this paper[22] draws conclusions which are in conflict with those of the author, a summary, supplemented by certain information not previously published, may be of interest.

The experimental equipment used by the writers was similar to that shown in Fig. 1 and about the same accuracy could be obtained with it. Nominal sizes for wrought-steel pipe were 2 in. and 3 in. The specific gravity of the mixture was determined by weighing measured volumes. The head loss was measured by differential manometers and expressed in feet of the flowing mixture. Runs were made with clear water before and after the series of runs with each sand. The test pipes were horizontal. During some of the runs, a 3-ft length of glass tubing of proper diameter was inserted in the pipe lines.

Conclusions pertinent to a discussion of Mr. Howard's paper were:

(1) At a given sand concentration, there was a certain velocity below which sand tended to accumulate on the bottom of the pipe, ultimately causing clogging if the flow conditions were maintained constant;

(2) Above this critical velocity, the head loss of the sand-water mixture,

NOTE.—The paper by George W. Howard, Jun. Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1938, by Messrs. Fred. R. Brown, Josef E. Montgomery, Elliott J. Dent, and David L. Neuman.

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[21a] Received by the Secretary November 9, 1938.

[22] "The Transportation of Sand in Pipelines," *Univ. of California Publications in Engineering*, Vol. 3, No. 7, November 12, 1937, pp. 343–384.

expressed in feet of the mixture, was found to be the same as for fresh water within the limits of accuracy of the experimental data; and,

(3) The ratio of the pressure drop (horizontal line) for clear water and sand-water mixtures was equal to the ratio of specific gravities.

If these conclusions are valid, they simplify the computation of head losses with sand-water mixtures because it is necessary only to know the friction loss in the same pipe with clear water. In order to test this relationship, the writers recomputed the published data of Miss Blatch,[23] W. B. Gregory,[24] N. Mikumo,[25] E. J. Dent,[26] M. Am. Soc. C. E., and unpublished data on a few dredges operated by the United States Engineer Department. It was found that all the laboratory experiments agreed with the conclusion that, above a certain critical velocity, the head loss for a sand-water mixture, expressed in feet of the mixture, was the same as the head loss for water in the same pipe. Below this critical velocity, the head loss remained very nearly constant as the apparent velocity, $\frac{Q}{A}$, was decreased. This phenomenon, observed first in the Blatch tests, was ascribed by the writers to accumulation of sand in the pipe to an extent which approximately held the true velocity constant. This conclusion was confirmed by the trend of the curves obtained by Professor Gregory on slurry so fine that it did not settle fast enough to reach equilibrium between runs.

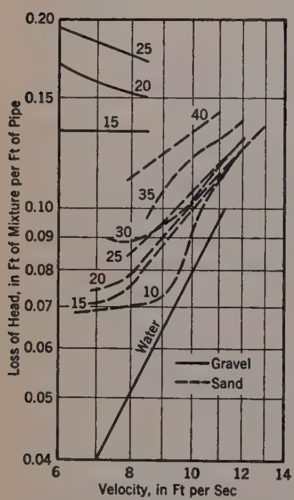

FIG. 11.—COMBINATION OF FIGS. 6(*a*) AND 6(*b*), EXPRESSED IN TERMS OF HEAD LOSS, IN FEET OF MIXTURE PER FOOT OF PIPE.

Fig. 11 shows all the author's data on sand and gravel mixtures and the friction curve for clear water. The trend of these curves is toward the "water" line; unfortunately, the experiments were not carried to sufficiently high velocities to determine whether the curves for mixtures are asymptotic. The data are in general agreement with the writers' conclusions previously mentioned.

There are a number of points in the paper under discussion which require clarification. In Fig. 4, the maximum velocity shown is approximately 6.3 ft per sec, whereas on the following page the velocity of flow is given as 8.88 ft per sec. The exact diameter of the test pipe before and after the experiments is not stated. Although the head loss with clear water is shown in Figs. 6(*a*) and 6(*b*), the friction loss or friction coefficient is not specified. The "water" curves in Figs. 6(*a*) and 6(*b*) appear to differ slightly, raising a

[23] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), pp. 400–408.

[24] "Pumping Clay Slurry Through a Four-Inch Pipe," *Mechanical Engineering*, Vol. 49, pp. 609–616.

[25] "On the Hydraulic Conveyance of Slime Pulps (Copper Ore)," *Journal*, Soc. of Mech. Engrs. of Japan, Vol. XXXVI, pp. 825–831.

[26] "Pipe Line Dredges," *Professional Memoirs*, Corps of Engrs., U. S. Army, Vol. VII, No. 31.

question as to whether this difference represents a real change in the pipe characteristic.

In introducing Equation (1), the author states that "when applied to water, exponential pipe formulas are more accurate than the Darcy formula * * *." There may be a question of convenience involved, but certainly not of accuracy if Equation (1) is properly used with f as an experimentally determined function of the Reynolds number.

Fig. 7, showing the lines of head loss as a function of velocity, is misleading. The points show a tendency (pronounced in the case of 30% of solids) to break away from the line, and extrapolation as shown is not justifiable.

The distribution of sand concentration shown in Fig. 4 may be explained at least quantitatively in terms of mechanism of turbulent flow. Field measurements by Colonel Dent[26] gave the same type of sediment distribution curve as Fig. 5. A conclusion drawn from this theory is that the settling velocity of the material being transported is its most important characteristic; the sieve analysis, specific gravity, percentage of voids, and other measurable characteristics are important only in so far as they provide a means of estimating the settling velocity. To complete the data, it is desirable that the author measure the mean settling velocity of the pea gravel and Pearl River sand used in his experiments. Using approximate methods based on the sieve analysis, the writers have estimated the mean settling velocities of the author's materials as: Sand, 0.19 ft per sec; and pea gravel, 0.76 ft per sec.

One of the obstacles to an understanding of the mechanism of sand transportation in pipe lines has been the failure to separate driving unit, pump, and pipe line in field tests. As a consequence, scores of over-all tests of dredges have been made without yielding data of general significance. A similar situation appears to exist in determining the economical operating conditions. The pipe line can, and should, be regarded as a separate unit requiring a certain power input in order to operate at a specified velocity and concentration. To this power may be added the loss in the pump, driving unit, transformers, and all other elements of the energy system, and the total power utilization may then be converted to terms of cost. To this cost, computed for a range of practicable operating conditions, must be added all over-head, interest, and other charges to obtain the total cost of the project. To be sure, some of these items are difficult to estimate, but they do not tend to make the pipe-line problem any less definite, as the author implies. As a matter of fact, the true situation seems to be that no one is really concerned about the economical velocity. Avoidance of clogging in non-uniform materials, specified dates for completion of the work, the fixed notions of operators, and other practical considerations control pipe-line velocities; but these do not make it impossible to come to some conclusion as to what the most economical velocity would be if one is interested in knowing it.

The experiments conducted by the writers were initiated for the purpose of comparing the difficulty of dredging in three locations. Representative samples were secured for the tests and the laboratory results had to be compared on a basis which would represent the relative dredging costs for the pipe line alone. For this purpose, the writers computed the work required to deliver

sand, per foot of pipe per pound of dry material. The resulting diagram for one sand appears in Fig. 12, from which it is evident that, considering the pipe line alone, the most economical velocity is the lowest obtainable without

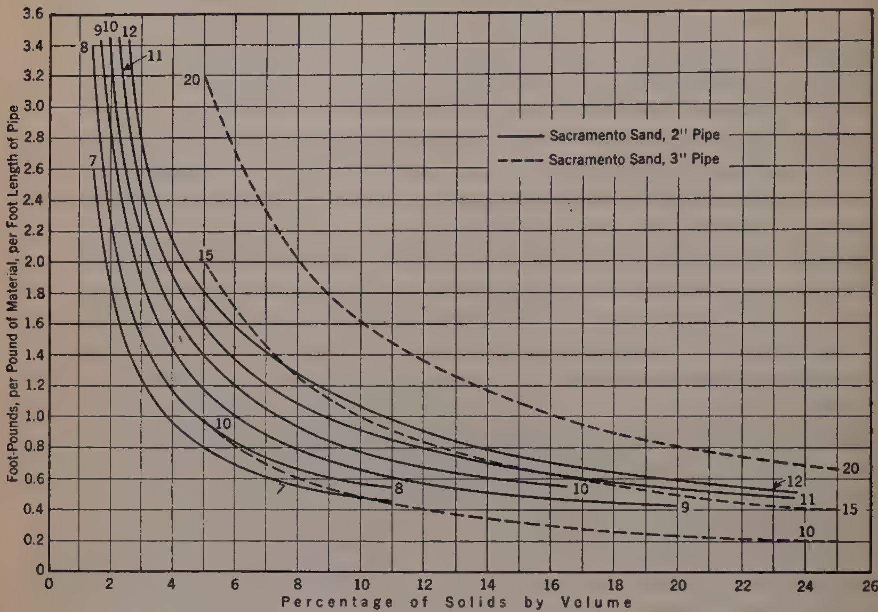

FIG. 12

clogging. Increases above this velocity are to be justified by other factors, and it should be possible in handling large quantities of uniform material to come to some reasonable conclusion as to the economical velocity.

Correction for *Transactions:* On page 2083, December, 1938, *Proceedings*, Line 24, change "∞" to read "00."

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DISCUSSIONS

MECHANICAL STRUCTURAL ANALYSIS BY THE MOMENT INDICATOR

Disscusion

By Messrs. John B. Wilbur, and William J. Eney

John B. Wilbur,[6] Assoc. M. Am. Soc. C. E. (by letter).[6a]—By the development of the moment indicator, the authors have not only made an important contribution to the available instruments for model analysis, but have opened the way for further development along similar lines. In the past, many different types of strain-gages have been developed for the measurement of linear strains, but the measurement of angular strains has not received as much attention. The moment indicator measures angular strains in such a manner that bending moments are easily obtained.

After describing the application of the moment indicator to the determination of moments in a Vierendeel truss model, the authors discuss several possible future developments of the instrument (see heading, "Further Considerations"), stating that a "valuable application of the method may be to the study of the so-called secondary stresses in trusses." They declare also that " * * * in some instances, it may be desired to ascertain the absolute magnitude of the bending moment at a point in the model, without having to measure the $N+1$ moments previously mentioned," and suggest that "this can be accomplished through the simple expedient of a 'spring balance' made of the same material as the model."

Subsequent to the presentation of the paper, further research under the direction of the writer has led to an investigation of these suggested applications. Since this investigation was based directly on the tentative suggestions made in the paper, a synopsis of the results obtained should be of interest.

The primary purpose of the later investigations was to determine, by means of the moment indicator and celluloid models, secondary bending moments in trusses with rigid joints. Three basic problems were present: First, it was necessary to devise a method of attaching the moment indicator to model

Note.—This paper by Arthur C. Ruge, M. Am. Soc. C. E., and Ernst O. Schmidt, Esq., was published in October, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

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[6a] Received by the Secretary November 29, 1938.

members so that its readings would not be influenced by shear distortions, because in truss models used for this purpose it is often necessary to use I-beams or their equivalent for members, the shear deflections of which are rather large, and impossible to determine accurately; second, it was necessary to magnify the readings of the moment indicator by a lever system, in order to insure sufficient precision of the microscope readings; and third, it was necessary to build and use a celluloid "spring balance," as illustrated in Fig. 9. Although the theory of the spring balance had been developed previously by Mr. Schmidt, it had not been tested by actual use. Equations of statics which are usually available for the conversion of relative values of moments (as measured by the moment indicator without the use of the spring balance) to actual values of moments due to the applied loads, are not present in dealing with the determination of secondary moments in trusses, so that it was essential that the spring balance be used.

An investigation by Mr. Johann Meier[7] showed that, for a rectangular beam, a transverse section which is plane before bending will, if not adjacent to points where external loads are applied to the beam, deform due to shear so that it takes the shape of a cubic parabola. Mr. Meier not only reached this conclusion from analytical investigations, but obtained an excellent experimental check working with a celluloid model. Near points of applied loads, however, longitudinal linear strains are affected by shear distortions, and the resulting variation from linear distribution in stresses normal to transverse sections causes shear distributions that are no longer parabolic, and shear distortions of transverse sections that are no longer in the shape of a cubic parabola. Because of the effect of Poisson's ratio, transverse linear stresses due to applied forces also affect strains normal to transverse sections, and, in turn, shearing distortions. Because of the uncertainty as to the shape which transverse sections will take as a result of shear distortion under varying circumstances, it was concluded that corrections to moment indicators, to allow for shear distortion effect, could at best be only approximate if the arms of the indicator were each attached to two points on a transverse section. Under no circumstances can the shear corrections be computed and proper corrections applied, when the moment indicator is attached to a member within a distance from the member support equal to the width of the member. When shear distortion is small, the errors due to this source are likely to be negligible, but in model studies for secondary moments in trusses, this may not be the case. It was believed highly desirable, therefore, to devise a method of attaching the moment indicator arms to members so that readings would not be influenced by shear.

After an extended investigation, it was shown by a model study that this could be accomplished for a moment indicator of the same dimensions as that described by the authors, by attaching an arm of the moment indicator to a member at two points 0.25 in. apart, both lying in the neutral axis of the member. Attached in this manner, it was found that readings were not only

[7] "An Experimental Check of the Theory of Shear Deformation and Shear Deflection," by Johann H. Meier. Presented in partial fulfillment of the requirements for the degree of Master of Science, at Mass. Inst. Tech., Cambridge, Mass., 1938.

uninfluenced by shear distortions, but that small errors due to other resulting sources were negligible. These negligible errors result from two causes: (1) The partial restriction of linear strains along the neutral axis between the two points of attachment; and (2) the fact that the arm of the indicator now is parallel to the chord joining the two points of connection to the elastic curve of the member, rather than tangent at a point half-way between these two points. The moment indicator designed for tests for secondary moments was built with this means of attachment, and it is believed that future indicators of this type should adopt this improvement. Simpler procedure as well as more accurate results may be obtained.

The rate of change of slope in members of a truss is not as large as it is in members of rigid frames without diagonals, so that a more sensitive moment indicator is required than would otherwise be the case. For this reason, a special moment indicator was built, which differed in two respects from that described in the paper. A lever system was inserted in the indicator which magnified the readings five times as compared to those which would otherwise have been obtained. Due to the presence of this lever system, it was desirable to omit the feature of the moment indicator which enables one to determine moments at both sections of attachment. In order to simplify the construction of the instrument, provision was made to determine moments at only one end of the indicator for a given setting on a member.

The practicability of the celluloid spring balance was first tested by measuring moments in a rigid frame without diagonals, using a celluloid spring balance in the line of loading. From a series of tests conducted in this manner, it was demonstrated that the idea could be used successfully, and that the accuracy of the moments thus obtained was comparable to that resulting from converting relative values of moments obtained without the spring balance, into actual values of moments, by the application of equations of statics. In the tests on celluloid trusses which followed, it was necessary to design the celluloid spring balances carefully, so as to make certain that they would be flexible enough to permit accurate readings, but at the same time strong enough so that they would not be over-stressed.

In a rigid frame without diagonals, where the loads are carried largely in bending, errors in model construction due to slight eccentricities in the members meeting at joints are not important, because the moments due to these eccentricities are small in comparison to the moments in the members meeting at the joint. Under these circumstances the moments in the ends of the members meeting at a given joint, as determined by the moment indicator, will be found to be nearly in static equilibrium. In trussed structures, however, eccentricity moments due to unavoidable, slight errors in model construction are relatively large in comparison with measured secondary moments. For this reason the moments measured at any joint may fail to be in static equilibrium by an appreciable amount. Therefore, a correction to the measured moments is necessary. This may be accomplished by distributing the unbalanced moments, as measured by the moment indicator, by the method of moment distribution, and combining the results of this analysis with the previously measured moments. An alternate procedure might be followed on

the model by applying moments at different joints, successively, and measuring the resulting moments set up in the members of the model, thus obtaining data to be used as a basis for corrections due to the eccentricity of members.

In comparing secondary moments as determined by model studies with analytical values for corresponding moments, an excellent agreement was found, with the exception of isolated cases in which, due to improper model design, the secondary moments were so large that the celluloid of the model was apparently stressed beyond its elastic limit. At such points, measured moments were appreciably lower than computed values.

It is believed that model studies of secondary stresses with the moment indicator are thoroughly practical, and that they offer an excellent approach to further investigations in the field of secondary stresses.

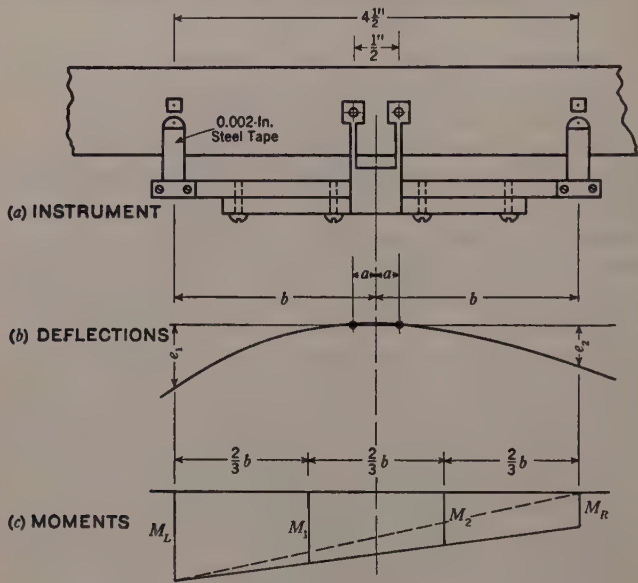

FIG. 10

It is of interest to consider types of moment indicators that differ in principle from that described in the paper. Perhaps the most direct method of determining bending moments in models is to measure the change of slope between two sections by means of two arms, normal to the member, each rigidly attached at points equidistant from the section where moment is to be determined. The measurement of the relative movements of points on the ends of the arms makes it possible to compute the bending moment by the moment-area theorem. This method was probably first devised by Professor Beggs, and has been used successfully under a variety of conditions. For use in celluloid models, however, it has been found that moment indicators, similar to that described by the authors, offer certain advantages, due mainly to simpler technique of application, and freedom from direct stress indicatiou superimposed upon the moment indication.

Several other types of moment indicators which differ in principle from that of the paper were designed in the Structural Analysis Laboratory in 1937. One of these designs, which will be referred to as Type II, holds considerable promise, and was actually constructed and tested. The theory of this type of moment indicator may be explained by reference to Fig. 10. In Fig. 10(a), e_1 and e_2 are the deflections of the ends of the instrument, measured relative to the chord through the two points of attachment to the beam, which are near the center of the instrument.

Deflections e_1 and e_2, in terms of the moments at the ends of the instrument, can easily be calculated, using, for example, the method of virtual work. They are:

$$e_1 = \frac{b^2 - a^2}{6\,E\,I}\,(2\,M_L + M_R) \qquad \text{(10a)}$$

and,

$$e_2 = \frac{b^2 - a^2}{6\,E\,I}\,(2\,M_R + M_L) \qquad \text{(10b)}$$

From the moment diagrams (Fig. 10(c)) it follows immediately, for the moments at one-third and two-thirds of the length of the instrument, that:

$$M_1 = \tfrac{2}{3}\,M_L + \tfrac{1}{3}\,M_R = \tfrac{1}{3}\,(2\,M_L + M_R) \qquad \text{(11a)}$$

and,

$$M_2 = \tfrac{2}{3}\,M_R + \tfrac{1}{3}\,M_L = \tfrac{1}{3}\,(2\,M_R + M_L) \qquad \text{(11b)}$$

Introducing these expressions in Equations (10):

$$e_1 = \frac{b^2 - a^2}{2\,E\,I}\,M_1 \qquad \text{(12a)}$$

and,

$$e_2 = \frac{b^2 - a^2}{2\,E\,I}\,M_2 \qquad \text{(12b)}$$

from which,

$$M_1 = \frac{2\,E\,I}{b^2 - a^2}\,e_1 \qquad \text{(13a)}$$

and,

$$M_2 = \frac{2\,E\,I}{b^2 - a^2}\,e_2 \qquad \text{(13b)}$$

Therefore, readings at the ends of the instrument are proportional to the moments at the one-third point and at the two-thirds point of its length.

Type II has not been as thoroughly investigated as the instrument described by the authors, but it is believed that it will give about the same accuracy of results. Its readings are not affected by shear distortions, and the labor involved in using the instrument may be somewhat less. Clamping the instrument to the center of a member rather than nearer its ends may be advantageous because of the effect of uncertain stress distribution in members near joints.

Type II is not mentioned for the purpose of comparing it with the moment indicator in the paper, but rather to illustrate the wide variety of instruments of this type that may be devised, and in this manner to bear out the thought that Professor Ruge and Mr. Schmidt have opened the way for important developments.

WILLIAM J. ENEY,[8] ESQ. (by letter).[8a]—In developing the moment indicator, the authors have made a very worth while contribution to the art of model analysis. As with all fundamental things, it is simple; and, as stated by the authors, it has many possibilities.

A structure composed of members with constant cross-section was wisely selected to serve as a demonstration of the method. The authors show that if the moment indicator is attached to a member of variable section, the appropriate slope deflection coefficients must be determined, either experimentally or by computations, before the moments are known. Since a large number of structures include variable section or curved members, the writer wishes to demonstrate a simple experimental procedure for treating such cases.

For determining these coefficients, use may be made of the apparatus shown in Fig. 11. This was originally designed to serve both in the determination of the elastic constants for the Cross moment-distribution method and as a deformeter gage for mechanical stress analysis. It is more elaborate than necessary for the purpose at hand. A large-scale model is made of that part of the frame model on which the moment indicator readings are taken and is attached to the gages, which are adjustable along a fixed aligning bar. End moments are applied by measured longitudinal pulls on the gage arms until the rotations of the arms are of the proper value. The effect of shear is automatically taken into account so that the true moments are obtained. In detail, the logic and procedure are as follows: In Fig. 12(*a*), the moment indicator is shown attached to a member with variable cross-section. Due to external loads on the structure, acting outside the gage length, the member

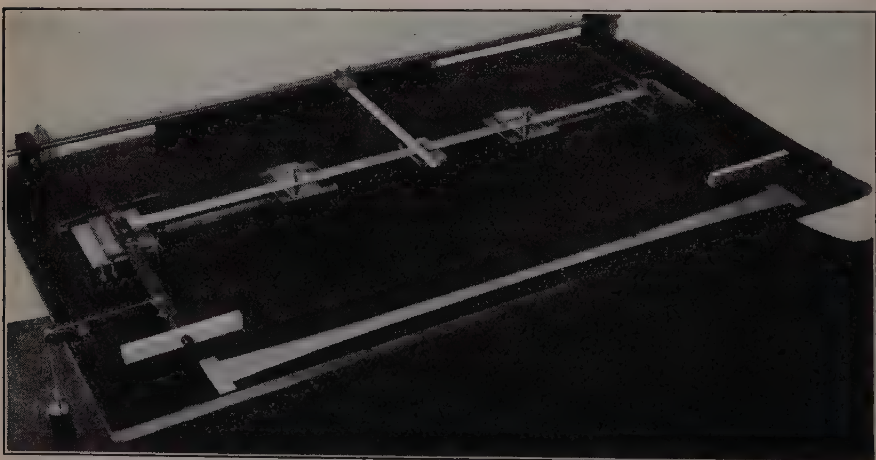

FIG. 11.—APPARATUS USED TO TEST MOMENT MODEL

deflects and the relative motions of the indices at the third points, y_1 and y_2, Fig. 12(*d*), are measured. The rotations of the elastic tangents at Points *A*

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[8a] Received by the Secretary November 30, 1938.

and B relative to the chord, $A\ B$, shown as Angles α and β on Fig. 12(d), are:

$$\alpha = \frac{y_4}{L} = \frac{2\,y_1 + y_2}{L} \quad \text{(14a)}$$

and,

$$\beta = \frac{y_3}{L} = \frac{2\,y_2 + y_1}{L} \quad \text{(14b)}$$

The distances, y_3 and y_4, are relatively small, compared with the gage length, L, and, therefore, the angle, in radians, and its tangent can be taken as equal.

The problem, then, is to determine what moments applied at Points A and B (Fig. 12(a)) of the frame model produce end rotations of α and β, respectively. It is not convenient to ascertain these values by applying moments to the frame model, nor to remove this section for testing and, later, to replace it by cementing with acetone. With such a small model, the application of the moments to produce the exact end rotations, α and β, would be tedious, and precision would be difficult to obtain. However, if this section of the frame model is reproduced to a larger scale such that it is 30 in. or 36 in. long, the end rotations can then be greatly increased, and the testing is a simple and practical matter.

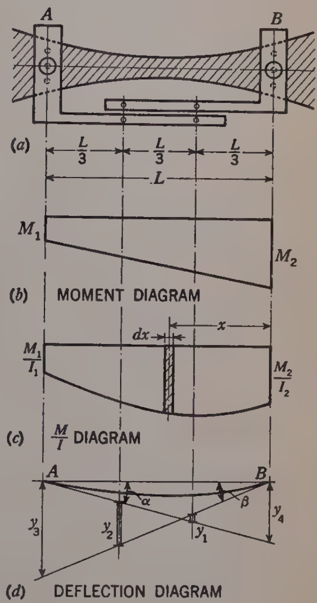

FIG. 12.—DIAGRAM ILLUSTRATING PRINCIPLE OF MOMENT INDICATOR.

Assume that this second model, which may be designated as the moment model, is proportioned so that the width scale is the same as in the frame model; but that the longitudinal scale is increased so that m-units of length in the moment model represent one unit of length in the frame model. The moment and frame models are cut from the same sheet material and, therefore, have the same modulus of elasticity. The $\frac{M}{I}$-diagrams in Fig. 12(c), for the two models subjected to numerically equal end moments, M_1 and M_2, would be similar in every respect since the moment of inertia at corresponding sections is equal.

The end rotation, α, for the frame model is the moment of the area under the $\frac{M}{I}$-diagram about Point B, divided by $E\ L$; that is,

$$\alpha = \frac{\int_B^A \frac{M}{I}\,dx\,x}{E\,L} \quad \text{(15a)}$$

Likewise, the rotation, α', for the moment model for the same end moments,

M_1 and M_2, is:

$$\alpha' = \frac{\int_B^A \frac{M}{I} (m\,dx)\,(m\,x)}{E\,(L\,m)} \quad \text{(15b)}$$

or the end rotation, α', of the moment model is m times larger than the rotation, α, of the frame model.

Sometimes it is desirable to use a different width scale for the moment model. For instance, if the moment of inertia at corresponding sections is represented by a width of 0.500 in. in the frame model and 0.250 in. in the moment model, the moment of inertia of the frame model is then eight times larger than that of the moment model at corresponding sections; hence, from Equations (15), $\alpha' = 8\,m\,\alpha$.

The precision can be increased if larger rotations are induced in the moment model; for instance, if α' and β' are both doubled, the frame moments, M_1 and M_2, will be one-half the moments applied to the moment model. The following example will further illustrate the method: The moment indicator of 4-in. gage length is attached to a member with a straight axis (see Fig. 12(*a*)). The width of the member varies but is symmetrical within the gage length. The relative moments, y_1 and y_2, of the indices at the third points (Fig. 12(*d*)) are zero and 0.0050 in., respectively. By Equations (14), the rotations, α and β, of the frame model are: $\alpha = \frac{0 + 0.0050}{4} = 0.00125$ radian; and, $\beta = \frac{2(0.0050) + 0}{4} = 0.00250$ radian.

The moment model is made 32 in. long, or eight times longer than the gage length of the frame model, and the width scale is unchanged. The corresponding end rotations to be induced in the model moment are: $\alpha' = m\,\alpha = 8 \times 0.00125 = 0.010$ radian; and, $\beta' = m\,\beta = 8 \times 0.00250 = 0.020$ radian. To increase the precision, rotations of $\alpha' = 0.020$ radian and $\beta' = 0.040$ radian will be used so that the frame moments will be one-half those of the moment model.

The moment model is rigidly clamped to the gage arms, as shown in Fig. 11. These arms pivot about a pin or ball-bearing and carry a pointer 10 in. from the pivot. The rotation is thereby magnified so that it is easily read on a scale of 100 divisions per in. The normal readings of both scales for the unstressed position of the moment model are recorded (see Fig. 13(*a*)) and then with thumb-screws the left arm at Point A is rotated clockwise 0.020 radian so that its pointer deflects 0.200 in. The right arm at Point B rotates freely 0.0135 radian (see Fig. 13(*b*)). A longitudinal pull must now be applied to the right arm to increase the counter-clockwise rotation, β', to 0.040 radian.

Instead of adjusting the pull until this exact rotation is secured, longitudinal pulls of a 1.54-lb weight acting 8 in. from the pivot, first on the pointer end of the right arm and then on the opposite end of the right arm, were applied. The right arm rotated 0.0235 radian counter-clockwise and then 0.0230 radian clockwise. (Fig. 13(*c*)), or an average rotation per inch-pound of moment of $0.0465 \div 24.62 = 0.00189$ radian. Therefore, to increase the rota-

tion of the right arm from 0.0135 to 0.040 radian a counter-clockwise moment of 14.00 in-lb is required (Fig. 13(*f*)). Consequently, a counter-clockwise moment of 7.00 in-lb acts on the frame model at the right end of the gage length.

The process is now reversed. The right arm at Point B is rotated 0.040 radian with thumb-screws and the left arm at Point A freely rotates 0.0278 radian (Fig. 13(*d*)). A longitudinal pull must now be applied so as to reduce the left arm rotation from 0.0278 radian to 0.0200 radian. As before, a longitudinal pull of 12.32 in-lb of moment acting first clockwise and then counter-clockwise causes a rotation of 0.0460 radian, or 0.0187 radian per in-lb of moment (Fig. 13(*e*)). Then, by proportion, a counter-clockwise moment of 4.17 in-lb decreases the rotation to 0.020 radian (Fig. 13(*f*)). The corresponding

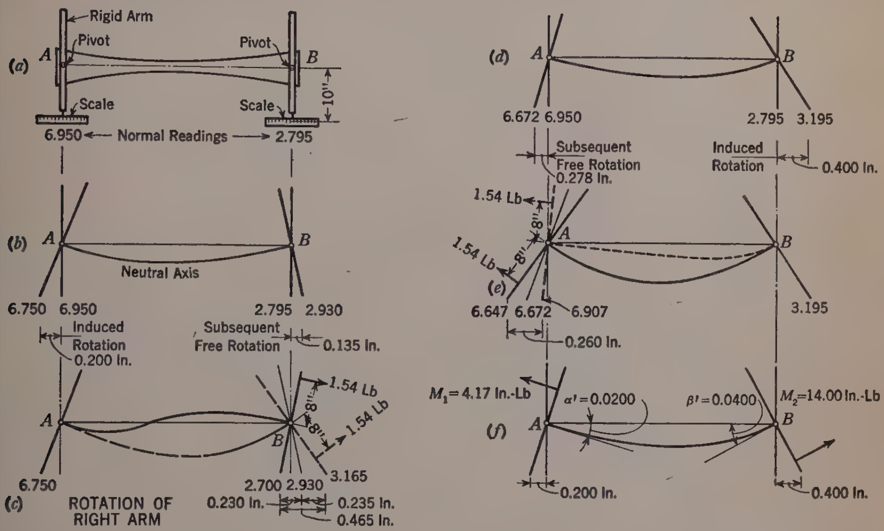

FIG. 13

moment, M_1, on the left end of the frame-model gage length is 2.08 in-lb, counter-clockwise.

The entire procedure is quite similar to the experimental determination of the elastic constants described elsewhere by the writer.[9] Note that no attempt is made to adjust the moments until the exact rotations, α' and β', are secured. In this way the tedious balancing of the weights simultaneously on each arm is eliminated. If the moment model has a curved axis the procedure followed in the foregoing example must be modified because the rotations due to the longitudinal pulls are dependent upon the kind of stress set up in the moment model.

The pivoting of the moment model so that in effect it is a hinged beam is a departure from perfect similitude. However, the writer has found that the model acts as if it were simply supported, and the end rotations are proportional

[9] "Fixed End Moments by Cardboard Models," by W. J. Eney, *Engineering News-Record,* December 12, 1935.

to the longitudinal pulls over a large range. If one is concerned with friction about the pivot or in the ball-bearing pulleys used for the longitudinal pulls, a constant-section moment model could be tested to calibrate the apparatus for all time.

By an obvious extension of the foregoing method, the moments can be determined for the case of loads acting within the gage length of the moment indicator. Some engineers will prefer the calculation of the moments, M_1 and M_2, to this simple experimental test. A summation process treating the diagrams as trapezoids, similar to the method proposed by George E. Large,[10] Assoc. M. Am. Soc. C. E., will expedite this work.

[10] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 101.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

STATE-WIDE SURVEYING PRACTICE IN MASSACHUSETTS

A SYMPOSIUM

Discussion

By Messrs. H. J. Shea, and Philip Kissam

H. J. Shea,[10] Jun. Am. Soc. C. E. (by letter).[10a]—In the face of such obstacles as had beset its path, the Massachusetts Geodetic Survey not only has triumphed, but has succeeded so well that the organization deserves a permanent place for the promulgation of engineering data in the Commonwealth of Massachusetts. Because of the haste with which the organization was instituted and the large number of parties operating in the field, the Home Office was shortly deluged with field observations. Recalling its early stages, it is not only a wonder that valuable data were forthcoming, but it is a marvel that the Massachusetts Geodetic Survey was able to extricate itself at all.

Massachusetts is indeed fortunate in having two excellent surveying agencies—the Massachusetts Geodetic Survey and the Land Court—both using the Lambert plane co-ordinates and advancing the engineering benefits of such a system. It is to be hoped that the Department of Public Works will be one of the first highway organizations to make widespread use of the Lambert co-ordinates. Such an application would obviate any duplication of data and would further efficient performance in this important public service.

However, the desired goal is not quite attained. Intelligent surveyors are after common applications of plane co-ordinates in point descriptions—those which concern the layman. The present problem is not one of "new surveying." No overhauling of specifications or methods of computations is necessary, but rather the problem appears to be the education of the every-day surveyor in the use of geodetic control through the translation of geodetic control into terms that are readily understood.

Note.—This Symposium was presented at the meeting of the Surveying and Mapping Division, Boston, Mass., October 7, 1937, and published in November, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the Symposium.

[10] Instr., Mass. Inst. Tech., Cambridge, Mass.

[10a] Received by the Secretary November 29, 1938.

Perhaps the educational necessity involved may be inferred from a statement about the condition of control surveys in Great Britain. In a personal communication, the Brigadier of the Ordnance Survey states that geodetic control cannot be considered limited because it covers the entire country, as do the published plans. All the surveys of Great Britain are conducted from that office. Its large-scale property plans are published at the scale of 25 in. to the mile. Quoting the Brigadier:

"They may need occasional addition and amplification by local authorities, but they are based on geodetic control and give graphically the positions of all important buildings and everything except the most recent developments in correct relative position. In England, the private surveyor is hardly ever called upon to take accurate measurements in the field."

Although the United States has no surveying unit similar in scope to the Ordnance Survey of Great Britain, the calculated effect of the present program should not be greatly different. Such a result may be obtained through unity of purpose in the various programs of surveying and the adoption of standard specifications.

In modern precise surveying, the importance of accurate geodetic control has become increasingly great. Scientific and well-planned surveying units, municipal or private, are quick to recognize the advantages derived from a uniform surveying control. By geodetic control is meant the establishment of surveying data, obtained either by precise triangulation, traverse, or levels, adjusted to remove accidental errors and based on a uniform datum. The standards of such work are considered so high that subsequent and more divided surveys are based upon, and adjusted to, the fixed values of the control survey.

There is one point in the problem of establishing control surveys that should be clearly understood. The surveyor is in the nature of a public servant and is often forced to deal with people having less specialized training. Although geodetic methods of computation should be the foundation, because of the difficulty in handling such computations, the greater number of surveyors are unable to use geographic values efficiently. The impartial observer, at present, finds it difficult to agree with impatient statements relative to the inability of surveyors to cope with the specialized methods of geodesy. It must be remembered, however, that the results of surveying operations are not confined to specialists only, but to lawyers, land owners, and dealers—in short, every one connected with land is intimately concerned therewith. Through the winning of the layman's confidence, the success of the present program will be assured. Thus, for a widespread realization of the benefits of control surveys, it is important to present the subject on a sound common basis rather than await the training of specialized surveyors.

A plane co-ordinate system is the apparent answer to the problem. In evaluating the relative advantages of the various co-ordinate systems, primary consideration should be given to the ease of reference to basic geodetic values. Although various systems have been suggested adapting co-ordinates to surveying, at present the most widely used are the tangent plane, the Lambert conformal projection, and the transverse Mercator projection co-ordinates. Co-ordinates on the latter two systems are handled much the same way, one

being adapted to States having greater east-and-west extent and the other to States having a greater north-and-south extent, respectively.

The characteristics of these systems may be compared in the following schedule, in which (*a*) refers to the use of the tangent plane, and (*b*) refers to the use of the Lambert and transverse Mercator projection:

Basic Origin.—
- (*a*) One local origin in the tangent plane. For wide use several tangent points are necessary (three are needed for New York, N. Y.).
- (*b*) The Lambert and transverse Mercator are based on two geodetic parallels (or a meridian). One State system covers States as large as North Carolina or New Jersey.

Comprehension.—
- (*a*) The tangent plane system is very easy to understand.
- (*b*) Somewhat more difficult than the tangent plane but can be grasped readily with some study.

Computation of Points and Traverses.—
- (*a*) Points are computed easily from geographic co-ordinates, but difficulty is encountered in traverses, because of the varying scales for which there is no definite mathematical treatment.
- (*b*) Points and traverses are computed simply. There is a definite relation between grid and geodetic lengths. Angles are well maintained through the rectangular grid.

Extent.—
- (*a*) Definitely limited and local.
- (*b*) State-wide; and basically national through ease of transfer from one system to another.

Basic Tables.—
- (*a*) The basic tables are excellent through the use of U. S. Coast and Geodetic Survey *Special Publication No. 71.*
- (*b*) Excellent through use of various State tables. Noteworthy of mention are the tables prepared in 1937 giving the plane co-ordinates of minute intersections of meridians of longitude and parallels of latitude published by the Massachusetts Geodetic Survey. This publication reduces the calculation of plane co-ordinates from geodetic points to mechanical interpolation.

Adjustment.—
- (*a*) Well adapted to adjustment, but dependability of adjusted values varies because of poorly determined scale effect.
- (*b*) Well adapted. Adjustments may be made directly with co-ordinate values on the grid. Subsequent geodetic values are easily found.

Publication of Co-Ordinates.—
- (*a*) Comprehensive list may be formed, but strictly local in scope.
- (*b*) State-wide publication possible.

At present, one disadvantage in the education of surveyors into the use of the Lambert and the transverse Mercator projection co-ordinates seems to be the necessity for the reduction of geodetic lengths to grid lengths. It

should be emphasized that the scale factors represent, not errors, but scale variations—mathematically exact in their relation to the geodetic values, free from any such stigma as "errors," easily applied to measured distances. Any inference of the existence of errors in the Lambert and transverse Mercator projections must be strongly rejected.

The legal aspect of geodetic control is perhaps one of the most potent factors in its favor. Land, certainly a common article of transfer, is often sold and bought without definite knowledge of the area, the protection afforded, and the surety that the deed description entitles the grantee to the property sometimes so vaguely described. Without attempting to draw "hard and fast" lines, the types of deed descriptions of property may be classified into four categories (seemingly in order of diminishing frequency); namely, land descriptions which, when without the original monuments or ties to the original monuments, are: (1) Not retraceable; (2) replaceable if a sufficient number of original monuments are recovered; (3) replaceable if one property monument is found or is replaceable because of ties to some public monument; and, (4) replaceable because description gives co-ordinates of property corners in some comprehensive system based on geodetic control.

Existing land rights in Case (1) are based on the best evidence possible, and two different surveyors might give widely different interpretations. Cases (2) and (3) depend for their interpretation on the recognition of extant bounds. In Case (4) replacement is simple if horizontal control points are within reasonable distances.

An important benefit resulting from Case (4) would be an amelioration of the ethical standards of the Surveying Profession. Too little emphasis has been placed on the legal ownership of surveys, a fact considerably abused by practicing surveyors. Property corners described in a co-ordinate system based on geodetic control would certainly leave no doubt as to the meaning of the description and should increase the confidence of the property owner in the work of the surveyor.

Hitherto, monuments have been thought to be the ultimate in boundary and property demarcation; but has not surveying experience in every section of the United States demonstrated that loss or displacement of monuments through construction and natural causes is sufficiently frequent to instigate well-placed skepticism in the lasting properties of physical markers? With a universal and comprehensive system of property description based on the required use of State-wide co-ordinates, does it not seem possible that there is at last a definite goal—a surveying element "better than the monuments"?

Mr. Houdlette's paper is noteworthy as an outline of one of the best surveying units of its kind. It must be regarded, therefore, as a valuable paper and would serve well as a guide for similar State organizations.

PHILIP KISSAM,[11] ASSOC. M. AM. SOC. C. E. (by letter).[11a]—The importance to the public of closely spaced, previously determined survey control points, and definitely determined, mutually controlled, and permanently located

[11] Associate Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

[11a] Received by the Secretary December 6, 1938.

boundary lines cannot be emphasized too strongly. This Symposium constitutes a description of an objective which must be ultimately attained throughout the United States. The advantages to be gained are obvious upon a moment's consideration.

It is evident that no survey is of use unless its precision is determined. Unless public survey control is available, individual control systems must be established for even the simplest project. If, on the other hand, standard, recognized control points of unimpeachable accuracy are established once and for all in a certain locality, all surveys made in that locality can be based on this control with the result that precision will be increased and survey costs reduced.

When once a survey has been completed, the data obtained cannot be combined with that of another survey unless both are based on the same system of control and preferably on the same datum. It is essential, therefore, that the control system, to be most effective, should be nation-wide so that in fact all surveys become part of one great survey. It will be noted that in the work that Mr. Houdlette describes, all control points, both vertical and horizontal, are connected with the national level, or triangulation net, so that the first requirement is abundantly fulfilled.

The utilization of the Massachusetts system of plane co-ordinates is an exceptionally important part of the project Mr. Houdlette describes. Practically all ordinary surveys are reduced to plane co-ordinates in order to determine their precision. It requires no more work or difficulty to use Massachusetts co-ordinates to make this reduction; but whenever Massachusetts co-ordinates are used for a survey, that survey is forever fixed in position and is correlated with every other survey so connected.

Perhaps the most important surveys of all are those made to define land boundaries. Property surveys are older than all others, and a large portion of the national wealth is dependent on their accuracy and permanence. Mr. Humphrey notes the set of rules he has worked out to be followed by land surveyors in making property surveys. Through these rules and by the able conduct of his office he has succeeded in developing a procedure and a type of land description which permanently locates land boundaries. Not only are the boundaries located definitely and permanently, but it is possible for any surveyor to re-mark the property corners by recourse to the description alone, and entirely without knowledge of local conditions. Those who are familiar with the archaic methods in common use to-day for making surveys and descriptions of land will recognize the singular importance of Mr. Humphrey's contribution.

One of the secrets of the success of the Massachusetts Land Court is the insistence of its engineer that durable monuments be established and that they be inter-connected by surveys so that they can be replaced when destroyed. Under his direction many local plane co-ordinate systems were developed, as noted by Mr. Houdlette. Quick to realize the importance of a universal datum, the Land Court now requires connection with the control points that the Massachusetts Geodetic Survey is establishing.

With this co-operation between the Land Court and the Massachusetts Geodetic Survey the cycle is complete. The Commonwealth of Massachusetts is being prepared for every type of survey requirement. It will be possible eventually to determine the relative position and elevation of every topographic feature or boundary line in the entire State with very little expenditure of time or money. It is evident that as these plans are developed, every survey made in the State will be correlated with the whole.

The Civil Engineering Profession cannot afford to overlook the advantages offered by this plan. If the profession is to take its proper place in public affairs every effort must be made to insure the establishment of these systems throughout the United States.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

LAND SURVEYS AND TITLES

FIRST PROGRESS REPORT OF THE JOINT COMMITTEE OF THE REAL PROPERTY DIVISION, AMERICAN BAR ASSOCIATION AND THE SURVEYING AND MAPPING DIVISION, AMERICAN SOCIETY OF CIVIL ENGINEERS

Discussion

By William Bowie, M. Am. Soc. C. E.

William Bowie,[1] M. Am. Soc. C. E. (by letter).[1a]—All those interested in the ownership and use of land will be encouraged by this report. The existing conditions so well described in the report are a reflection on the common sense of the American people. The report shows how matters may be improved; and surely some remedial action will soon be taken by the Federal Government, the governments of States, cities, counties, and by private corporations and individual property owners.

Many comments appear in the press, and in other journals, on the necessity of finding new ways of using the intelligence and efforts of the people in order to reduce unemployment. This is a most worthy objective. Whether the answer will be to start some great new industry or a number of small ones can be learned only with time. Those engineers and members of the legal profession, however, who deal with the ownership of land feel that the time has arrived for putting many people to work on property, or what is generally called cadastral, surveying.

Twenty years or more ago the recommendations of the Committee would have created only an academic interest. The horizontal control surveys of the United States had not been extended to the point that final geographic positions to the corners of property could have been determined except for a few places.

Note.—This First Progress Report of the Joint Committee of the Real Property Division, American Bar Association, and the Surveying and Mapping Division, American Society of Civil Engineers, on Land Surveys and Titles, was published in November, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[1] Hydrographic and Geodetic Engr., U. S. Coast and Geodetic Survey (*Retired*), Washington, D. C.

[1a] Received by the Secretary December 10, 1938.

At present, there are thousands of miles of arcs of accurate triangulation in the United States, and the triangulation net can be completed within a decade, or sooner, if that is desired. Plane co-ordinate systems have been devised for all of the States, which make it possible to use the triangulation stations as the bases of local property surveys without the engineer or surveyor having to use the troublesome spherical co-ordinate system.

What is needed now is a comprehension on the part of the officials of the governing bodies of the United States that the recommendations of those who are able to point the way to better methods should be given earnest consideration—something that has not been done except in a few instances.

A good beginning toward carrying out the recommendations of the Committee could be made in each of the cities and large towns of the United States because the triangulation has been already extended to the area of most of the large settlements of the nation. The city should organize a control survey branch of its engineering department for the purpose of supplementing the existing triangulation. In some cases it will be necessary to do more triangulation to serve as the base for the traverse that will be designed to supply many closely spaced stations for use in boundary surveying. The methods for doing triangulation are well developed and can be followed without difficulty by any engineer who has good judgment and eyesight. It would be well for an engineer of the city government to spend a few weeks with a triangulation party of the U. S. Coast and Geodetic Survey in order to learn the best ways of organizing a party and in making the observations, setting monuments, erecting the observation towers, etc. All this can be learned from textbooks and reports, but much time, money, and effort can be saved by actually viewing the operations in the field by those who have had years of experience.

The triangulation must be supplemented by accurate traverse surveys which must be made by the local authorities. It is essential that the fundamental triangulation net be completed by the Federal Government. This work is necessary as the control for topographic mapping (a Federal project), but the property boundary surveying is a local problem. If, in each city, one or more traverse parties could be put in the field it would not be long before control stations would be available to those who might be engaged in boundary surveying. This also applies to the counties. The cost of such work would be small in comparison with the savings that would result. In order that the local control surveys might be made with accuracy and at low cost the instrumental equipment should be of the best. It is not economy to save on the instruments and add much to the operating cost of the surveys. Engineers to-day use the most modern equipment on their construction work, but they are rather slow to change to modern surveying instruments. It is reasonably certain that good instruments would save their original cost in one year. A transit can be used for twenty years or more with only occasional servicing, so the saving on the pay-roll during such a period would be many times the cost of the instrument.

It is very desirable that the survey or mapping office recommended in the report be created in each State and in each of its political subdivisions, in order that some supervision may be given to the control surveys and to the

boundary surveying. Slip-shod methods should no longer be tolerated. Such offices would be in a position to furnish information on the control survey data that would avoid expensive duplication of effort.

It would be desirable for each State to pass acts that would lead to the safeguarding of survey monuments of all kinds, and that would provide that monuments may be established on private property, and the engineer and surveyor may enter private property for the purpose of using existing monuments. Of course, provision should be made in the act for the compensation of the owner, should any damage to his property result from its use.

Owing to the great importance of making property boundary surveys with accuracy, and that they be well monumented, it is most desirable that all public control surveys be made in such a way that they can be incorporated into the national horizontal control net. Tens of thousands of miles of traverse have been made by branches of the Federal Government, but in only a few cases are the surveys of such an accuracy as would justify their use in cadastral surveying. This is notably true of the work done by the Topographical Branch of the U. S. Geological Survey and by the Corps of Engineers of the U. S. Army. The officials of these organizations claim that the acts of Congress calling for certain work did not provide that the surveying be done for any but the immediate purpose. The officials hold, and justly so, that they cannot divert any of the funds to surveying that would cost more than those of a character just accurate enough for their projects. One can sympathize with an official who is requested to do a large amount of work with a small sum of money; but it would be well to have a comprehensive surveying and mapping plan for the United States which would lead to the co-ordination of effort on the part of all who make surveys. What has been said of the work of the Federal agencies applies also to the agencies of States. The surveys made in connection with the location and construction of highways, in general, are of such nature that they do not fit into any comprehensive survey of the United States. Nothing can be done about the lack of accuracy and also lack of monuments of past surveys; but in the interests of the people satisfactory work should be required on all future control surveys of a public character. Supposedly, only self-interest on the part of taxpayers and property owners will bring about the improvement.

It would be of great value if the rights of way of public property and property of private corporations could be resurveyed and monumented, and the co-ordinates of the monuments determined. The monuments would be useful in other cadastral surveying along the routes, and the owners of adjoining property would be free from the uncertainty that exists in many cases when the rights of way are not well and accurately defined.

Even where there are no triangulation stations available, the boundary surveys should be made with requisite accuracy, and the corners should be monumented. Later, as triangulation is extended to the area, connections could be made to the existing monuments and co-ordinates computed for the property corners.

The writer has stressed the surveying phases of the report; but it is of equal importance that means be provided that will safeguard the owner of land by

having issued to him a title that will be so definite that no doubt of the boundaries would arise; and that there will be no uncertainty as to liens on the property. The record should be so complete that both the owner and any one who may wish to acquire land would never be in any doubt. The transfer of title would be easier and less costly than at present if the recommendations of the Committee should be adopted.

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DISCUSSIONS

SOLUTION OF EQUATIONS IN STRUCTURAL ANALYSIS BY CONVERGING INCREMENTS

Discussion

BY A. FLORIS, ESQ.

A. FLORIS,[6] ESQ. (by letter).[6a]—An ingenious method for solving a certain type of simultaneous linear equation by successive approximations is presented by the author. In these equations the major coefficients (that is, those belonging to the principal diagonal of the matrix) are appreciably greater than the other coefficients. A more refined criterion of these equations has been given by E. von Mises and H. Polaczek-Geiringer.[7]

The method of converging increments is in reality the mathematical counterpart of the statical analysis of moment distribution. It is exactly the same method as that developed by J. Morris in 1935.[8] The difference between the two is slight and consists mainly in the manner of presenting the subject and the arrangement of the equations. However, the speeding up of the calculations by series summation in case of slow converging equations is shown only by Professor Dell.

The method of converging increments is not an improvement over ordinary iteration.[9] The principal advantage in the use of iteration lies in the fact that, in each step or cycle, an approximate value of the unknowns is determined, which can be improved further by repeating the process. This important advantage can be realized in the author's method only by additional labor. For this purpose the underlined values must be added at the end of each cycle and then divided by the corresponding major coefficients in order to obtain the values of the unknowns. Unless these checks are made, errors that creep in during the distribution will remain unnoticed. Such intermediate calculations are not necessary in the ordinary method of iteration.

NOTE.—The paper by George H. Dell, Assoc. M. Am. Soc. C. E., was published in October, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[6] Dipl.-Ing., Los Angeles, Calif.

[6a] Received by the Secretary November 9, 1938.

[7] "Praktische Verfahren der Gleichungsauflösung," von E. von Mises und H. Polaczek-Geiringer, *Zeitschrift für angewandte Mathematik und Mechanik*, February, 1929, p. 58.

[8] "On a Simple Method for Solving Simultaneous Linear Equations by a Successive Approximation Process," by J. Morris, *Journal*, Royal Aeronautical Soc., April, 1935, p. 349.

[9] "Analysis by Moment Distribution Aided Through Use of Iteration," by A. Floris, *Engineering News-Record*, June 25, 1936, p. 922; or "Shear Deformation Included in Three-Moment Equation," by A. Floris, *Civil Engineering*, October, 1937, p. 711.

Inasmuch as the simultaneous equations encountered in structural analysis are usually very sensitive, the method of converging increments, in certain cases, may make the process extremely lengthy or divergent. The criterion for such cases is given by Mr. Morris.[8]

The assumed values of the unknowns in the ordinary iteration are corrected step by step, until the continuation of the process is rendered superfluous. Errors can easily be detected, and if they are overlooked they retard the solution to be sure, but they do not render the calculations worthless, as is the case with other methods. For equations arising in statics, the cycles of repetition will rarely exceed five or six. The method is simple and easy to apply; and, its range of application is wider.

With regard to the examples chosen by the author, it may be pointed out that the simultaneous equations of Example 1 could be solved equally well, if not better, by the method of successive elimination of unknowns, using an ordinary 10-in. slide-rule. Such cases are rather rare in practical problems.

In conclusion, it may be stated that the method of converging increments is interesting no doubt, but it cannot replace ordinary iteration, as far as problems in statics are concerned.

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DISCUSSIONS

RECONSTRUCTION OF A PIER IN BOSTON, MASSACHUSETTS, HARBOR

Discussion

By William G. Atwood, M. Am. Soc. C. E.

William G. Atwood,[3] M. Am. Soc. C. E. (by letter).[3a]—One possible method of reconstruction, used in some other piers in Boston, Mass., is not mentioned by Professor Spofford, and it would be interesting to know whether it was considered. Reference is made to the use of treated timber piles to replace the damaged piles. This would have made it necessary to remove the rip-rap but that might not have been costly enough to make this method uneconomical as compared to that used. By the use of timber-pile replacements the entire reconstruction of the deck might have been avoided.

The successful driving of piles of such great length is a tribute to the care and preparation by the engineers and contractors.

It may seem strange that the original designers of the pier did not protect the piles against the attack of marine borers, but the reason is clear to those who know the conditions in Boston Harbor. The investigation of 1922–1923 did not disclose the presence of borers in sufficient numbers to indicate that there was any danger from them. There were piers in the harbor that dated back to Colonial days, and there was no indication that they had ever been attacked; and there was no record of serious damage to any structures. Except for some of the railroad piers there was practically no treated timber in the harbor. The conditions in New York (N. Y.) Harbor are practically the same to-day, except that there is a history of attack by borers in the past.

The cause or causes of the invasion of borers into Boston Harbor is not known since there seems to have been little change in the water conditions. The cause of the heavy attack in the Harbor of Lynn, Mass., on the other hand, followed the removal of the sewage from the harbor and the consequent return of the normal oxygen content to the water.

Note.—The paper by Charles M. Spofford, M. Am. Soc. C. E., was published in October, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[3] Cons. Engr., Ithaca, N. Y.

[3a] Received by the Secretary November 1, 1938.

There seems to be little doubt but that the protection of the substructures from attack by marine borers is cheap insurance. The cost of reconstructing two wharves in Boston Harbor, one of which[2] is described in this paper, would probably pay for the protection of at least 150 000 piles of normal length. Many other wharves have been rebuilt as a result of this attack and most of the remainder will have to be rebuilt eventually.

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[2] See, also, *Civil Engineering*, December, 1937, p. 843.

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DISCUSSIONS

ENERGY MASS DIAGRAMS FOR POWER STUDIES

Discussion

BY EDGAR E. FOSTER, ASSOC. M. AM. SOC. C. E.

EDGAR E. FOSTER,[4] ASSOC. M. AM. SOC. C. E. (by letter).[4a]—The usefulness of the energy mass diagram for solving problems of power supply is well presented in this paper. As the author states, the idea of the energy mass diagram is derived by analogy from the mass flow diagram or curve; but, by substituting rates of energy or power for the stream flow the utility of the diagram is extended considerably. It may then be applied to any form of power generation, steam and pumped storage, as well as to hydro-electric power.

The basis of the extended utility may be easily inferred from a simple equation expressing the mass flow diagram, as follows:

$$B = \int_0^t Q\,dt \qquad (2)$$

in which B is the volume of water obtained at a given point in time, t, and Q is the rate of flow. For Q and B, similar quantities of other elements may be substituted. As the author has done in effect, E may be used for the total output of energy from a power system and e for the rate. Then,

$$E = \int_0^t e\,dt \qquad (3)$$

Integrating, $E = e\,t$, from which $e = \frac{E}{t}$, which is the average rate over the time, t. Expressing the equation in another form,

$$\frac{dE}{dt} = e \qquad (4)$$

that is, the tangent at any point equals the rate.

NOTE.—The paper by John W. Hackney, Jun. Am. Soc. C. E., was published in October, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] Assoc. Engr., U. S. Engr. Office, Washington, D. C.

[4a] Received by the Secretary December 7, 1938.

The writer used this type of diagram several years ago in a study to calculate the maximum prime power in a co-ordinated system of several reservoirs and run-of-river plants. Although the answer sought was only an approximation, the quantities and graph will serve very well to describe the process.

Thirteen proposed and existing plants were included in the system. There were five reservoirs with a total capacity of 75.4 billion cu ft of useful storage. On the assumption that the uppermost reservoir would be drawn down first and the procedure repeated for each successive reservoir down stream, and that the water was utilized at full head on the plants below, the energy equivalent of the storage was 0.843×10^9 kw-hr.

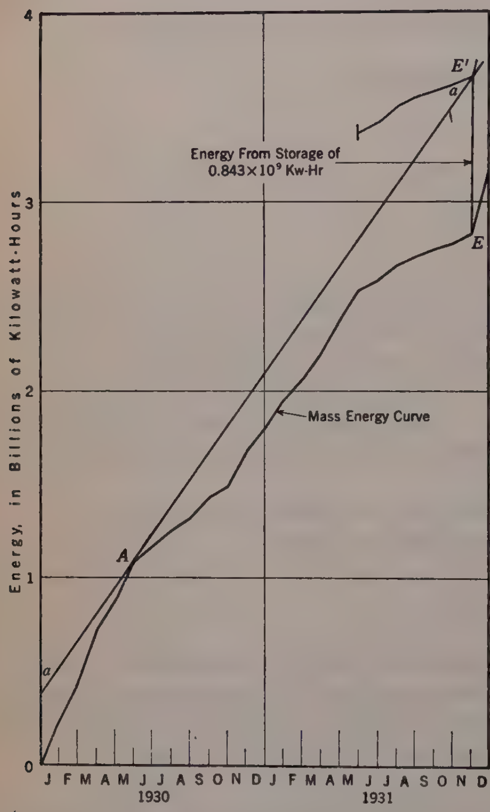

FIG. 3.—COMMON TANGENT ON PARALLEL MASS DIAGRAMS

From previous study it was known that the critical period of the stream flow was that from January 1, 1930, to December 31, 1931. The problem, therefore, consisted in the determination of the maximum rate of prime power obtainable through that period with the given system of power plants.

The energy mass curve used in solving this problem in Fig. 3 as Line $O\ A\ E$ for the period from January 1, 1930, to December 31, 1931. Following its construction, a part of the same curve was plotted parallel to itself at a distance of 0.843 billion kw-hr, as shown at Point E'. The entire curve was not plotted parallel because, as will readily be seen, only the critical parts need be repeated. A tangent, $A\ E'$, was then drawn from Point A on the main curve (at which point the reservoirs were assumed to be full) to Point E' on the upper segment. This completed the construction of curves.

The slope of this tangent, $A\ E'$, is the maximum prime power obtainable through that period of record. Point E' is the sum of the total power from natural run-off plus the regulated flow from storage. At Point E' all the reservoirs are empty and the maximum energy, therefore, has been obtained for the period. The time of E and E' is identical. Then,

$$E' = e'\,t \qquad (5)$$

in which e' is the rate of output from the time of Point A to the time of Point E', or,

$$e' = \frac{E'}{t} \qquad (6)$$

which equals the maximum uniform rate of prime power. With Equation (6), e' is found from the slope of Tangent $A\,E'$. From Point a to Point a', the time is approximately 25.2 months, or 766 hr, and the total energy output is 3.6×10^9 kw-hr; or $e = \frac{3.6 \times 10^9}{766} = 197\,000$ kw, which is the maximum rate of prime power.

The author has constructed his energy mass diagram by using the differences between the monthly output of power and the mean for the period. The construction of the mass diagram from residuals does have an advantage (as the author has stated) in bringing into sharper prominence the critical points of a mass curve. The usual depressions caused by deficiencies of natural stream flow are filled in to some extent by the energy equivalent of storage, when a mass curve of energy is constructed from the output of a system of power plants, including reservoirs.

This method of construction, however, does not change the fundamental principles. If $\bar{e}$ is the mean rate for the period, the mass diagram may be represented by the equation,

$$E = \int_0^t e\,dt - \int_0^t \bar{e}\,dt \qquad (7)$$

Since this is true, the mass curve of residuals (or only the critical segments) can be plotted above the parallel to itself at a distance denoting the energy equivalent of the storage, and a common tangent can then be drawn to determine the maximum prime power.

By constructing two complete curves, at a distance representing the energy equivalent of stored water, the studies of operation can be readily conducted by means of the tangents. Any rate of power output can be introduced by simply changing the slope of the tangent. All that is necessary is that the tangent stay within the parallel mass curves.

Referring now to Fig. 2, it appears to the writer that the auxiliary diagram could be eliminated by simply limiting, in the calculations, the power output of Plant R to the normal rate—namely, 210 000 kw. By crediting Plant R with power below this rate instead of 280 000 kw as a maximum, the "dump" power or waste would not be included in the upper diagram and, therefore, no correction would be necessary as at Point G.

Errors of this method of power analysis are not serious enough to be objectionable. In the event that the errors arising from monthly means are too great, shorter periods of time, as a week or even a day, can be taken. The critical periods are relatively short and excessive work is not entailed by the use of shorter increments of time through such periods to refine the calculations as much as is consistent with the data. The errors of the process can be reduced by this means to limits within the errors of the original stream-flow and power-load data.

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DISCUSSIONS

GREAT LAKES TRANSPORTATION

Discussion

By T. Kennard Thomson, M. Am. Soc. C. E.

T. Kennard Thomson,[4] M. Am. Soc. C. E. (by letter).[4a]—The French and then the English managed to reach Lewiston and Queenston, on the Niagara River, by water transportation with a few land portages, between Montreal and Lake Ontario (St. Lawrence River).

They encountered the first formidable portage, a climb of more than 300 ft and a land haul of about 30 miles, from Lewiston to near Buffalo. This afforded the main route for trade (chiefly with Indians) until the Canadians constructed the Welland Canal. The construction of the Erie Canal was due to the untiring and unselfish efforts of De Witt Clinton (1769–1828) who contributed his time and money without stint. No one else did nearly as much for the project. He started the construction of the Canal in 1817, and completed it in 1825.

The Erie Canal, as then built, was 4 ft deep, 40 ft wide, and was used by boats of from 30 to 70 tons. By 1825 it had earned its entire cost of $7 143 789, and, at the same time, greatly reduced the freight rates, thereby enormously increasing the population and wealth of the State. By 1862 the Canal had been deepened to 7 ft for boats of 240 tons. The enlargements from 1837 to 1862 cost $31 834 041. Work was started to increase the depth to 9 ft, but it was not completed.

The cost to 1899 was $48 000 000. In 1881 all tolls on the Canal were abolished. Prior to that time the net revenue had exceeded all costs by $42 000 000. The Barge Canal was started in 1904 and completed to a depth of 12 ft in 1918, at a cost of $170 000 000. Prior to 1908 the revenue from the Canal had been about $360 000 000, or $20 000 000 more than the total cost to that date.

Under the heading, "Shipping: New York State Barge Canal-Boats," General Tyler states that "because of the peculiar physical conditions to be met, a distinct type of boat has been developed." The writer presumes that

Note.—The paper by M. C. Tyler, M. Am. Soc. C. E., was published in the November, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] Cons. Engr., New York, N. Y.

[4a] Received by the Secretary November 20, 1938.

he refers to the limited height under the bridges, which make it a "Barge Canal."

The reason for changing the Erie Canal to a "Barge Canal" was because the politicians were afraid that New York, N. Y., and Buffalo, N. Y., would vote against the proposed enlargement, if ships could pass through the Canal without being unloaded and loaded at Buffalo and New York. Hence a "Barge Canal" was constructed, with fixed bridges that make it impossible for any boat standing 15.5 ft above the surface of the water to pass under the bridge.[5]

Another mistake made on the Barge Canal was in not relocating the line near Rome, N. Y., so that it would be unnecessary for all boats from Buffalo to "lock up" about 50 ft, before "locking down" again. As it is, water from Lake Erie cannot reach the Hudson River, and it was necessary to create great artificial lakes, by dams at Delta, N. Y., Hinckley, N. Y., etc., to supply water for the Canal east of the hump.

For many years there has been talk about constructing a ship canal from the Hudson River to Lake Ontario or Lake Erie, so that ocean-going vessels could reach the Great Lakes. Before the depression (1930) it was estimated that such a ship canal would cost $650 000 000. As the Americans can use the new Canadian Welland Canal—the best route for the ship canal would be from the Hudson River to near Oswego, N. Y., on Lake Ontario.

General Tyler states that the entrance to Buffalo Creek was improved in 1826, and that the Government breakwater in front of Buffalo Creek was constructed in 1869. (See heading, "Channels and Harbors," Improvements (6) and (7).) As Buffalo still has a very inadequate harbor, the writer has, for many years, been advocating the reclamation of from 6 to 10 sq miles from Lake Erie, just outside the Government breakwater, for a great railroad and shipping terminal.

As Buffalo will eventually extend from Lake Erie to Lake Ontario, it will then have two ports for ocean-going vessels, one at Elevation 242, and the other at Elevation 576, above sea level.

As the author states (see heading, "Channels and Harbors"), the enlargement of the Welland Canal, with a depth of 25 ft of water was completed in 1932. Many wonder why this well-constructed canal was built to such a depth before the St. Lawrence River was developed, when boats drawing less than one-half that depth of water could not reach Lake Ontario from Montreal, and could not enter many of the Lake ports. Some say that it was due to "keeping sub-election promises."

As stated in the paper, the important commerce over the Barge Canal now consists of coal, ores, wheat, etc. In the earlier days it was mostly for Indian trade which was for many years controlled by the Great Hudson Bay Company.

It is now claimed that ores can be delivered in New York Harbor, *via* the Lakes and the Barge Canal, at a smaller freight charge than they can be delivered in Pittsburgh.

The indispensable test of all construction should be the question: "Will it pay?" The only way in which this condition can be met in developing the

[5] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 867.

navigation facilities of the St. Lawrence River, to permit ocean-going vessels to reach Lake Superior, will be by developing the water power of the Niagara and St. Lawrence Rivers.

For twenty-seven years, the writer has been advocating, a dam ("Niagara Falls Junior") to be located about 4.5 miles down the river from the Old Falls, or below the Whirlpool.[6] For about the same time, he has also been advocating the reclamation of about 10 sq miles from the St. Lawrence River, between Lachine and Montreal, Que., Canada, and the development of 1 500 000 hp, hydro-electric, at that site.

If these projects are constructed economically, as they should be, industries will be attracted sufficient to create a dozen Pittsburghs along the banks of the Niagara and St. Lawrence Rivers, bringing great financial prosperity to the entire continent of North America and benefitting many other parts of the world.

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[6] "Niagara Falls, Jr.," an interview with T. Kennard Thomson by G. P. Roney, *The Banker and Financier*, September 15, 1921.